**PREAMBLE**

We are very pleased to introduce the Book of Abstracts of the XXVII Congress of Differential Equations and Applications and XVII Congress of Applied Mathematics (XXVII CEDYA/XVII CMA), which collects the abstracts of all the contributions involved in the Congress.

The Spanish Society of Applied Mathematics (SeMA) was created in 1991, as a contribution in the development of Mathematics and its applications in a wide range of scientific and industrial areas. Among SeMA main purposes one can find the development of scientific computation and numerical simulation, mathematical modelling and analysis and control techniques. The society organizes congresses, publishes the SeMA Journal and the SeMA Bulletin Newsletter, awards the best scientific contributions to the SeMA Journal, recognizes the best young researcher with the Antonio Valle Prize, participates in dissemination activities, ... 

SeMA has its origin in a series of conferences on Differential Equations and Applications (CEDYA). These conferences began as the initiative of a group of Spanish researchers, most of them from the areas of Applied Mathematics and Mathematical Analysis, with the goal of boosting the research in Differential Equations, Numerical Analysis, Mechanics, Optimization and Control. The first edition was held in El Escorial (Madrid) in 1978 and organized by the Complutense University of Madrid. After this first edition, the congress was held annually until 1987. Over the course of time, the congress topics were updated to incorporate new areas of Applied Mathematics. Because of this evolution, the congress was renamed to Congress on Differential Equations and Applications/Congress of Applied Mathematics (CEDYA/CMA) in the 1987 edition. After this edition, the conference changed to be biennial and SeMA was officially founded in the 1991 edition.

The Congress holds its 27th meeting at the Faculty of Medicine of the University of Zaragoza from July 18th to 22nd, 2022, organized by the Institute of Mathematics and Applications (IUMA) and the SeMA and with the collaboration of the Faculty of Sciences.

The aim of CEDYA/CMA is to encourage mathematical researchers from all over Spain and the world to present their novel research results and their latest developments. This book of abstracts gathers together both novel research results of high impact for the mathematical community and provide an invaluable resource for many researchers. We do hope that you will enjoy reading those abstracts of particular interest in your field.

In this edition, CEDYA/CMA will feature eight plenary lectures by internationally renowned researchers in mathematics, eighteen minisymposia proposed by different researchers and groups, and seven thematic sessions organized by the local organizing committee to distribute the individual contributions. Altogether, the congress consists of more than 250 communications (talks and posters) and more than 280 participants.

We look forward to seeing you in the next CEDYA/CMA.

CEDYA/CMA 2022 Local Organizing Committee
ACKNOWLEDGEMENTS

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CONFERENCE PROGRAM

PLENARY TALKS

2 Reduced order models for incompressible flows
   Julia Novo

3 Numerical simulation of flow problems in deformable porous media
   Carmen Rodrigo

4 Hypocoercivity-preserving Galerkin discretisations
   Emmanuil Georgoulis

5 Dynamics of excitable cells: neurons and cardiomyocytes
   Roberto Barrio

6 From linear to nonlinear n-width: optimality in reduced modelling
   Albert Cohen

7 Port-Hamiltonian systems: a new paradigm for modeling, simulation and control of complex systems
   Volker Mehrmann

8 Asymptotic preserving schemes for hyperbolic systems of balance laws
   Giovanni Russo

11 Optimal control in soft robotics
   Francisco Periago

TALK AWARD WINNERS

13 Theory and methods for random differential equations: An overview and some recent results
   Marc Jornet

14 Concentration versus Simplification in Aggregation-Diffusion Equations
   David Gómez-Castro

15 Some open problems in dynamical systems
   Armengol Gasull

16 Numerical analysis of some nonlinear hyperbolic systems of Partial Differential Equations arising from Fluid Mechanics
   Ernesto Pimentel-García
P1. Partial Differential Equations

18 Existence of solution for a class of semilinear problem with a unknown Radon measure data
Juan Francisco Padial

19 Existence and regularity of solutions in a semilinear problem with singularity in the datum
José Carmona Tapia, Antonio J. Martínez-Aparicio, Pedro J. Martínez-Aparicio, Miguel Martínez Teruel

20 Convergence of weak solutions of semilinear elliptic problems with datum in L1
José Carmona Tapia, Antonio J. Martínez-Aparicio, Pedro J. Martínez-Aparicio, Miguel Martínez Teruel

21 A nonlocal model of Hyperelasticity and vector calculus for (one-point) nonlocal gradients over bounded domains
José Carlos Bellido, Javier Cueto, Carlos Mora-Corral

22 Weak boundary conditions for Boussinesq flow with free out-flow, Part I: Analysis
Malte Braack, Dandy Rueda Castillo

23 Degenerate Kirchhoff-type elliptic equations with convective reaction
Calogero Vetro

24 Fractional Laplacian BVP with nonlinearities having multiple zeroes
Rubén Fiñana Aránega, José Carmona Tapia

25 Concentration phenomena in the Lifshitz–Slyozov system with inflow boundary conditions
Juan Calvo, Erwan Hingant, Romain Yvinec

26 Stieltjes Parabolic Partial Differential Equations. Application to a Digital Twin for a population dynamics problem
Francisco J. Fernández, Iván Area Carracedo, Juan J. Nieto Roig, F. Adrián F. Tojo

27 A general spatial predator-prey model
Julién López-Gómez, Eduardo Muñoz-Hernández

28 Lie symmetries and exact solutions for a fourth-order non-linear diffusion equation
Almudena del Pilar Márquez, Tamara María Garrido, Elena Recio, Rafael de la Rosa

Posters

29 Predictive ROM based on a coordinate transform technique applied to 1D and 2D transport problems
Pablo Solán-Fustero, José Luis Gracia, Adrián Navas-Montilla, Pilar García-Navarro

30 A priori estimates for a 1–Laplace evolution equation
Marta Latorre Balado, Sergio Segura de León
P2. DYNAMICAL SYSTEMS - ORDINARY DIFFERENTIAL EQUATIONS

32 Traveling waves for Non-degenerated Reaction-Diffusion equations with a possibly bounded flux
Juan Campos

33 Hopf Bifurcation for a Functional Differential Equation (FDE) with respect to the delay
Alfonso C. Casal, Juan Francisco Padial

34 Asymptotic behavior of solutions to lattice diffusion equations
Antonín Slavík

35 Matrix versions of higher-order recurrence relations for Sobolev-type orthogonal polynomials
Carlos Hermoso, Edmundo J. Huertas, Alberto Lastra, Francisco Marcellán

36 D-concave nonautonomous scalar bifurcation theory and applications in critical transitions
Jesús Dueñas, Carmen Núñez, Rafael Obaya

37 Lie algebra of linear colored network dynamical systems
Fahimeh Mokhtari, Jan Sanders

38 Control properties of multiagent singular systems under a geometrical point of view
M. Isabel García-Planas

39 Autonomous and non-autonomous unbounded attractors in evolutionary problems
Piotr Kalita, Juan García Fuentes, Jakub Banaśkiewicz

40 About the structure of attractors for a nonlocal Chafee-Infante problem
Rubén Caballero, Alexandre Carvalho, Pedro Marín-Rubio, José Valero

41 Mutlichromatic Travelling Waves for Lattice Nagumo Reaction Diffusion Equation
Hermen Jan Hupkes, Leonardo Morelli, Petr Stehlík, Vladimír Švígler

42 Large Scale Optimization using Multiresolution Techniques
Rosa Donat, Sergio Lopez-Ureña

43 Oscillation-based Numerical Integrators for HMC
Fernando Casas, Luke Shaw, Jesús-María Sanz-Serna

Posters

44 Reduced dimensionality model for sprint kayak propulsion: applications
Diego Delgado, Camilo Ruiz

45 Chaos in the Two-Coupled Brusselators Model: Building a Bridge in the Literature
Ana Mayora-Cebollero, Fátima Drubi, Jorge A. Jover-Galtier, Álvaro Lozano, Carmen Mayora-Cebollero, Lucía Pérez, Rubén Vigara

46 Structure of Non-Autonomous Attractors for a Class of Diffusively Coupled ODE
Alexandre Carvalho, Luciano Rocha, José Langa, Rafael Obaya

47 Runge-Kutta-Nyström methods for the numerical solution of second order linear inhomogeneous IVPs
Manuel Calvo, Juan Ignacio Montijano, Luis Rández
P3. Numerical Analysis and Simulation

50 High order PPH schemes over nonuniform grids
   Isabel Jiménez, Pedro Ortiz, Juan Ruiz, Juan Carlos Trillo

51 Generalized Weighted Power Means. Properties and uses
   Sergio Amat, Isabel Jiménez, Juan Ruiz, Juan Carlos Trillo, Dionisio F. Yáñez

52 Splines cúbicos monótonos para datos procedentes de funciones discontinuas
   Francesc Aràndiga, Antonio Baeza, Dionisio F. Yáñez

53 On error estimations for multiscale hybrid-mixed finite element methods on polyhedral meshes
   Sônia Gomes

54 An improved model for the drying of solids with abruptly changing porosity
   Maria González, Hiram Varela

55 Integration of the source term in transcritical rarefactions when using approximate Riemann solvers
   Juan Mairal, Javier Murillo, Pilar García-Navarro

56 An a posteriori error analysis for incompressible elasticity problem with mixed boundary conditions
   Tomás Barrios, Marcelo Behrens, Rommel Bustinza

57 New families of eighth-order Runge–Kutta–Nyström symplectic integrators
   Alejandro Escorihuela-Tomàs

58 Numerical analysis of a poro-elastic MGT problem
   Noelia Bazarra, José R. Fernández, Ramón Quintanilla

59 Evaluation of a general car-following model for micro/macro traffic modelling
   Juan Francisco Padial, José Enríquez

60 Boundary integral methods for simulating scattering and transmission problems in 2D elastodynamics
   Víctor Domínguez, Catalin Turc

61 Solving linear and nonlinear PDEs in the exascale using Monte Carlo
   Jorge Morón, Francisco Bernal, Andrés Berridi, Juan Acebrón, Renato Spigler

62 Meshless finite difference method for solving fractional differential equations at irregular meshes
   Antonio M. Vargas

63 A deep neural network for solving PDEs using r-adapted meshes
   Á. Javier Omella, David Pardo

64 Adapting cubic Hermite splines to the presence of singularities through correction terms
   Sergio Amat, Zhilin Li, Juan Ruiz-Álvarez, Concepción Solano, Juan C. Trillo
65 A finite difference approximation scheme for the self-adjoint extensions of singular Sturm-Liouville operators
Victor Laliena

Posters

66 Numerical computation of the fractional Laplacian with $\alpha = 1$ by means of Gauss hypergeometric functions
Carlota María Cuesta, Francisco de la Hoz, Ivan Girona

P4. NUMERICAL LINEAR ALGEBRA

68 Totally positive matrices, Bernstein polynomials, and applications to CAGD
José-Javier Martínez, Javier Martínez Yubero

Posters

69 On the best approximation of square matrices in Laplacian-like form
J. Alberto Conejero, Antonio Falcó, María Mora

P5. OPTIMAL CONTROL - INVERSE PROBLEMS

71 Optimal control for cardiovascular diseases
Jorge Tiago, Irene Marín Gayte

72 Imaging electromagnetic scatterers in the time domain
Timo Lähivaara, Peter Monk, Virginia Selgas

73 Null controllability for linear parabolic equations. Applications to therapies
Inmaculada Gayte

74 Comparison of the topological derivative behavior on different scenarios: the time-harmonic heat and Maxwell equations
Ana Carpio, Manuel Pena, María-Luisa Rapún

75 Numerical methods for object detection in attenuating media
Ana Carpio, María-Luisa Rapún

Posters

76 Parabolic ejection & collision orbits for the restricted planar circular three body problem
José Lamas Rodríguez, María Teresa Martínez-Seara Alonso, Marcel Guardia Munarriz

77 Bernstein-type Operators based on the Jacobi inner product
David Lara Velasco, Teresa E. Pérez
P6. APPLIED MATHEMATICS TO INDUSTRY, SOCIAL SCIENCES AND BIOLOGY

79 A mathematical model for the energy stored in green roofs
Francesc Font, Maria Aguareles, Marc Calvo-Schwarzwalder, Timothy G. Myers

80 A degenerating convection-diffusion system modelling froth flotation with drainage
Raimund Bürger, Stefan Diehl, MCarmen Martí, Yolanda Vásquez

81 Time parallel methods for epidemiological models
Noelia Ortega Román, Victoria Redondo Neble, José Rafael Rodríguez Galván

82 Computing $\Delta V$ for orbital maneuvers between non-coplanar and non-coaxial elliptical Keplerian orbits
Ángeles Dena, Elvis Lacruz

83 Bow appendages in small fishing vessels. Resistence analysis
Aurelio Muñoz Rubio, M. Victoria Redondo Neble, Alexis Sánchez Abadía

84 Microwave irradiation and conventional heating: a comparison using in silico experiments with water and ethylene glycol
Maria Cruz Navarro, Damián Castaño

85 Model transform and local parameters. Application to instantaneous attractors
Javier A Galadí, Fernando Soler-Toscano, Jose A Langa

86 Wind modeling applications: a case of study applied to the estimation of electricity production in wind farms
José Miguel Pablos-Marín, Diego Prieto-Herráez, María Isabel Asensio, José Manuel Cascón-Barbero

87 An Inextensible Model for the Robotic Manipulation of Textiles
Franco Coltraro

88 Improving mathematical simulations through high quality input data: a case of study applied to the simulation of wildfires
Marcos López-De-Castro, Diego Prieto-Herráez, María Isabel Asensio, Gianni Pagnini

90 Immersed boundary approach to biofilm spread on surfaces
Rafael González-Albaladejo, Ana Carpio

91 Models and numerical methods for pricing renewable energy certificate derivatives
María A. Baamonde-Seoane, María-del-Carmen Calvo-Garrido, Carlos Vázquez

92 Coupled models for equilibrium problems with heterogeneous agents
Jonatan Ráfales, Carlos Vázquez

Posters

93 Bayesian mechanistic model of COVID-19 transmission dynamics including the effect of vaccination
Javier Blecua, Juan Fernández-Recio, José Manuel Gutiérrez
Towards large-scale sustainable water photo-electrolysis: modelling the electrochemical behaviour of titanium dioxide electrodes

J.C. Ciria, Ricardo Celorrio, Alejandro Ansón-Casaos, Carlos Martínez-Barón, Ana M. Beníto, Wolfgang K. Maser

**P8. OTHER: SCIENTIFIC CALCULUS, APPROXIMATION THEORY, DISCRETE MATHEMATICS**

On the Motion of Two Point Masses inside a Homogeneous Cloud

Luis Floría

A subdivision scheme to refine piecewise-smooth data on triangular meshes

Conti Costanza, Sergio López-Ureña

**Posters**

The Christoffel function on and in quadratic revolution surfaces

Gema Alhama, Miguel Piñar

**MS01. DYNAMICAL SYSTEMS: THEORY AND APPLICATIONS**

Organized by Sergio Serrano, Fernando Fernández-Sánchez, María Ángeles Martínez and Óscar Rodríguez

Central configurations of planar restricted (4+1)–body problems with some equal masses

Montserrat Corbera, Jaume Llibre, Claudia Valls

Arnold Diffusion in the Restricted Planar Elliptic Three Body Problem

Marcel Guardia, Jaime Paradela, Tere Seara

Dynamics of a non-symmetric dipole-segment asteroid model

Eva Tresaco, Patricia Sánchez-Martín

Semi-analytical computation of heteroclinic connections between center manifolds with the parameterization method

Miquel Barcelona, Alex Haro, Josep-Maria Mondelo

Invariant tori around the Moon including the solar gravitational effect

Ángel Jorba, Begoña Nicolás

A dynamical definition of the Sphere of Influence of the Earth

Irene Cavallari, Clara Grassi, Giovanni F. Gronchi, Giulio Baù, Giovanni B. Valsecchi

Chaotic coorbital motions to $L_3$ in the Restricted Planar Circular 3-Body Problem

Inmaculada Baldomá, Mar Giralt, Marcel Guardia
Splitting and coexistence of any number of strange attractors in families of expanding baker maps
A. Marqués-Lobeiras, A. Pumariño, J. Á. Rodríguez, E. Vigil

Splitting of separatrices for rapidly forced pendulum with a perturbation without first harmonic
Román Moreno, Inmaculada Baldomá, Tere M. Seara

Scattering in the elastic collision of He atoms off a corrugated Cu surface
Esther Barrabés Vera, Florentino Borondo, Ernest Fontich , Pau Martín, Mercè Ollé

Ejection/collision orbits and beyond
Mercè Ollé

Ejection/collision orbits for the hydrogen atom in a circularly polarized microwave field
Esther Barrabés, Mercè Ollé, Óscar Rodríguez

Resonances in the hydrogen atom
Aitor Santamaria, Marc Jorba-Cuscó, Daniel Pérez-Palau

Metapopulation interaction models for infectious diseases
Marc Jorba-Cuscó, Daniel Pérez-Palau

On the persistence of the tripod gait
Roberto Barrio, Álvaro Lozano, Sergio Serrano, Ana Mayora, Carmen Mayora, Rubén Vigara

Optimal control of neural populations for Communication Through Coherence
Antoni Guillamon, Gemma Huguet, Michael Orieux

Coriolis coupling in a Hénon-Heiles system
Víctor Lanchares, Manuel Iñarrea, José Pablo Salas, David Farrelly

Computable normal forms for piecewise smooth systems with a pseudo-focus
Marina Esteban, Emilio Freire, Enrique Ponce, Francisco Torres

Dynamic Hopf-like bifurcation in piecewise linear systems
Mathieu Desroches, Jordi Penalva, Catalina Vich, Antonio E. Teruel

Limit cycles of polynomially integrable discontinuous piecewise linear differential systems
Belén García, Jaume Llibre, Jesús S. Pérez del Río, Set Pérez-González

Limit cycle bifurcation from infinity in 3D relay systems
Emilio Freire, Enrique Ponce, Javier Ros, Elísabet Vela

Zero-Hopf bifurcation at infinity in 3D piecewise linear systems with symmetry
Enrique Ponce, Javier Ros

Birth, transition and maturation of canard cycles in PWL systems
Víctoriano Carmona, Soledad Fernández-García, Antonio E. Teruel

The Hindmarsh-Rose model: classification of spike-adding processes and bifurcation structure
Roberto Barrio, Santiago Ibáñez, Lucía Pérez, Sergio Serrano

Analysis of travelling waves in cortical spreading depression models involving multiple time scales
Gemma Huguet, Tere M-Seara, Carles Bonet, David Reyner-Parra
Ordered structures of chaotic invariant sets on square-wave neuron models
Sergio Serrano, M. Angeles Martínez, Roberto Barrio

The Poincaré map of degenerate monodromic singularities with Puiseux inverse integrating factor
Isaac A. García, Jaume Giné

Invariant algebraic curves for certain generalized Liénard differential system via Puiseux integrability
Jaume Giné

Period function of planar turning points
Renato Huzak, David Rojas

Dissecting bifurcation diagrams: geometric bifurcations
Roberto Barrio, Santiago Ibáñez, Lucía Pérez

Lie point symmetry algebras of some quadratic differential systems: dynamical consequences
Isaac A. García, Susanna Maza

Bifurcations in the Riemann ellipsoids
Fahimeh Mokhtari, Jesús F. Palacián, Patricia Yanguas

Fast-slow analysis in cardiac dynamics
Roberto Barrio, Jorge Alberto Jover-Galtier, María Ángeles Martínez, Ana Mayora-Cebollero, Lucía Pérez, Sergio Serrano

MS02. NUMERICAL APPROXIMATION OF HYPERBOLIC PDE SYSTEMS AND THEIR APPLICATIONS

Organized by Tomás Morales de Luna, Cipriano Escalante Sánchez and José Garres Díaz

Minimally implicit methods for the resolution of the neutrino transport equations
Samuel Santos-Pérez, Martin Obergaulinger, Isabel Cordero-Carrión

High order numerical schemes for generalized models in chromatography
Rosa Donat, María del Carmen Martí, Pep Mulet

Integrating functions with singularities: adapted quadrature rules
Sergio Amat, Zhilin Li, Juan Ruiz-Álvarez, Juan Carlos Trillo, Concepción Solano

Well-balanced implicit-explicit Lagrange-Projection scheme for the one-dimensional shallow water system
Celia Caballero Cárdenas, Manuel J. Castro Díaz, Tomás Morales de Luna, María de la Luz Muñoz Ruiz

Arbitrary high order well-balanced WENO finite volume scheme using Flux Globalization
Mirco Ciallella, Davide Torlo, Mario Ricchiuto
Implicit and semi-implicit high-order well-balanced methods for one-dimensional systems of balance laws
Sebastiano Boscarino, Manuel Jesús Castro Díaz, Irene Gómez-Bueno, Carlos Parés, Giovann Russo

In-cell Discontinuous Reconstruction path-conservative methods for non conservative hyperbolic systems - MOOD extension
Ernesto Pimentel-García, Manuel Jesús Castro, Christophe Chalons, Tomás Morales de Luna, Carlos Parés

High-order well-balanced finite volume schemes for 1d and 2d shallow water equations with Coriolis forces
Víctor González-Tabernero, Manuel Jesús Castro-Díaz, Jose Antonio García-Rodríguez

Numerical simulation of a Fractional Flow Model for Oil-Water Movement in Porous Media
Arturo Hidalgo, Paula Luna

Analysis And Numerical Simulation Of 2D Shallow Water Moment Equations
Julian Koellermeier, Rik Verbiest

Modelling and dispersion relations of layer-averaged non-hydrostatic Euler equations
Cipriano Escalante, Enrique D. Fernández-Nieto, José Garres-Díaz, Tomás Morales de Luna, Yohan Penel

An efficient IMEX-DG solver for the compressible Navier-Stokes equations for non-ideal gases
Paolo Barbante, Luca Bonaventura, Giuseppe Orlando

Global entropy stability for a class of unlimited second-order schemes for hyperbolic systems of conservation laws
Ludovic Martaud, Christophe Berthon, Mehdi Badsi

A staggered semi-implicit hybrid finite volume / finite element scheme for the shallow water equations at all Froude numbers
Saray Busto, Michael Dumbser

Well-Balanced High-Order Discontinuous Galerkin Methods for Systems of Balance Laws
Ernesto Guerrero Fernández, Cipriano Escalante, Manuel J. Castro Díaz

A family of hybrid finite volume / finite element schemes for computational fluid dynamics
Laura del Río Martín, Saray Busto Ulloa, Michael Dumbser

Modelling of dry granular flows with weakly non-hydrostatic pressure
Enrique D. Fernández-Nieto, José Garres-Díaz, Tomás Morales de Luna, Anne Mangeney

A general multilayer-moment based approach for the vertical approximation of free-surface Euler equations
José Garres-Díaz, Cipriano Escalante, Tomás Morales de Luna, Manuel J. Castro Díaz

A first-order hyperbolic reformulation of the Navier-Stokes-Korteweg system
Firas Dhaouadi, Michael Dumbser
MS03. PDE MODELS IN BIOLOGY

Organized by María Ángeles Rodríguez-Bellido and Cristian Morales-Rodrigo

157 Numerical schemes for attraction-repulsion chemotaxis systems
Silvia Frassu, José Rafael Rodríguez-Galván, Giuseppe Viglialoro

158 An Upwind DG Scheme Preserving the Maximum Principle for the Convective Cahn-Hilliard Model
Daniel Acosta-Soba, Francisco Guillén-González, J. Rafael Rodríguez-Galván

159 A non-local free boundary problem arising in a model of cell polarization
Anna Logioti, Barbara Niethammer, Matthias Röger

160 Nonlocal elliptic system arising from the growth of cancer stem cells
Manuel Delgado, Ítalo B. M. Duarte, Antonio Suárez

161 Diffusive Lotka-Volterra models including nonlocal intraspecific and interspecific interactions
William Cintra, Mónica Molina-Becerra, Antonio Suárez

162 Structural Dynamics and Indirect Effects in the Lotka-Volterra Model
José David Gutiérrez de Alba

163 On a comparison method for a parabolic-elliptic system of chemotaxis with density-suppressed motility and logistic growth
J. Ignacio Tello

164 Continuous and discrete periodic asymptotic behavior of solutions to a competitive chemotaxis PDEs system
Mihaela Negreanu

165 First results for a bilinear optimal control problem for the Keller-Segel model with logistic term
Pablo Braz e Silva, Francisco Guillén-González, Cilon F. Perusato, María Ángeles Rodríguez-Bellido

166 Dual Discrete Finite volume scheme for degenerate parabolic equations
Mazen Saad, El Houssaine Quenjel, Ben Mansour Dia

167 Finite element technique for chemotaxis phenomena based on shock capturing
Santiago Badia, Jesús Bonilla, Juan Vicente Gutiérrez-Santacreu

168 On the numerical study of a chemotaxis system modeling tumoral invasion
Viviana Niño-Celis, Jhean E. Pérez-López, Diego A. Rueda-Gómez, Élder J. Villamizar-Roa

169 Theoretical and numerical analysis for a model of glioma invasion
Jorge L. Lópe-Agredo, Diego A. Rueda-Gómez, Élder J. Villamizar-Roa

170 On a chemotaxis-reaction PDE-ODE system modeling glioblastoma
Antonio Fernández-Romero, Francisco Guillén-González, Antonio Suárez

171 Analysis of an attraction-repulsion chemotaxis model with saturated signals
Silvia Frassu, José Rafael Rodríguez Galván, Giuseppe Viglialoro

172 Criteria towards boundedness for attraction-repulsion Keller–Segel systems
Silvia Frassu, Giuseppe Viglialoro
Numerical schemes for a chemo-attraction and consumption model
Francisco Guillén-González, Giordano Tierra

MS04. OPTIMAL CONTROL AND INVERSE PROBLEMS

Organized by Mariano Mateos

Optimization problems related to antiangiogenic therapy for tumor growth models
Luis Alberto Fernández, María Peña

Observability and control of parabolic equations on networks
Jone Apraiz, Jon Asier Bárcena-Petisco

Multilevel control
Umberto Biccari, Deyviss Jesús Oroya Villalta, Carlos Esteve-Yagüe, Enrique Zuazua

On a special class of boundary optimal control problems
Pablo Pedregal

Nonlocal basis pursuit: Nonlocal optimal design of conductive domains in the vanishing material limit
José Carlos Bellido, Anton Evgrafov

Topology optimization problems with connectivity constraints
Ernesto Aranda, Alberto Donoso, David Ruiz

Topology optimization of flexoelectric materials considering a micromorphic approach
Rogelio Ortigosa, Jesús Martínez-Frutos, Antonio J. Gil

Load matrix recovery from scattering data in linear elasticity
Juan Antonio Barceló, Carlos Castro, Mari Cruz Vilela

Exponential turnpike property for fractional parabolic equations with non-zero exterior data
Mahamadi Warma, Sebastián Zamorano

Acoustic Full-Waveform Inversion via Optimal Control: First- and Second-Order Analysis
Irwin Yousept

Inverse problem for Hamilton-Jacobi equations: projections onto the reachable set and semiconcave envelopes
Carlos Esteve Yagüe
MS05. Iterative Processes and Non Linear Equations

Organized by José M. Gutiérrez, Sergio Amat, Miguel Á. Hernández-Verón and Jean-Claude Yakoubsohn

187 Familia de métodos iterativos de resolución de sistemas no lineales basada en el método de Homeier
Francisco I. Chicharro, Alicia Cordero, Neus Garrido, Juan R. Torregrosa

188 Métodos iterativos para la obtención simultánea de raíces de ecuaciones no lineales
Alicia Cordero, Neus Garrido, Juan R. Torregrosa, Paula Triguero-Navarro

189 Extensión del análisis discreto multidimensional para métodos vectoriales con memoria
Alicia Cordero, Neus Garrido, Juan R. Torregrosa, Paula Triguero-Navarro

190 Chebyshev-type methods for the matrix p-th root
Sergio Amat, Sonia Busquier, Miguel Ángel Hernández-Verón, José Antonio Ezquerro, Natalia Romero

191 Funciones generadoras de Catalan
Pedro J. Miana, Natalia Romero

192 Generalization of the secant method. High order methods free of derivatives
Vicente Candela, Natalia Expósito, Pedro J. Martínez-Aparicio, Rosa Peris, Juan Carlos Trillo

193 Iterative method for multiple roots without knowing multiplicity
Alicia Cordero, Juan Ramón Torregrosa, Paula Triguero-Navarro

194 Some progress in the study of Schröder’s method applied to cubic polynomials
Víctor Galilea Martín, José Manuel Gutiérrez Jiménez

195 New results on the dynamics of the Chebyshev-Halley family of iterative methods
José M. Gutiérrez

196 Iteration of rational Hopf-endomorphisms for graphical representation of basins of attracting n-cycles
Víctor Álvarez, José Manuel García, Luis Javier Hernández, María Teresa Rivas

197 High order method for the SVD
Jean-Claude Yakoubsohn

198 An unfeasibility view of neural network learning
Joos Heintz, Luis Miguel Pardo, Enrique Carlos Segura, Hvara Ocar, Andrés Rojas Paredes

MS06. ALAMA: Linear Algebra, Matrix Analysis and Applications

Organized by Ana Marco, José-Javier Martinez and Raquel Viaña

200 Gaussian quadrature formulae and total positivity
Ana Marco, José-Javier Martínez, Raquel Viaña
201 Linear algebra over linear control systems
    Miguel Carriegos

202 On bundles of matrix pencils under strict equivalence
    Fernando De Terán, Froilán M. Dopico

203 High relative accuracy for some subclasses of $P$-matrices
    Jorge Delgado, Héctor Orera, Juan Manuel Peña

204 Accurate computations with totally positive Lagrange-Vandermonde matrices
    Ana Marco, José-Javier Martínez, Raquel Viana

205 Accurate computations with Bernstein mass matrices
    Esmeralda Mainar, Juan Manuel Peña, Beatriz Rubio

206 Totally Positive Matrices and Gaussian Markov Random Fields
    Juan Baz, Pedro Alonso, Juan Manuel Peña, Raúl Pérez-Fernández

207 Backward stability in rational eigenvalue problems solved via block Kronecker linearizations
    Froilán Dopico, María C. Quintana, Paul Van Dooren

208 Compensation of positivity in the Symmetric Nonnegative Inverse Eigenvalue Problem
    Carlos Marijuán, Julio Moro

209 Tuned preconditioners for linear systems associated with the time-dependent neutron $S_N$ equations
    A. Carreño, D. Ginestar, L. Bergamaschi, A. Martínez, A. Vidal-Ferràndiz, G. Verdú

**MS07. NEW TRENDS ON THE 1-LAPLACIAN**

*Organized by Alexis Molino and Marta Latorre*

211 The 1-Laplacian in Metric Graphs
    José M. Mazón

212 Nonlocal nonlinear diffusion problems with nonlinear boundary conditions
    Marcos Solera Diana

213 Anisotropic Chan-Vese segmentation
    Salvador Moll, Vicent Pallardó

214 Weak solutions to the total variation flow in metric measure spaces
    Wojciech Górny

215 On the first Robin eigenvalue of the $p$-Laplacian as $p$ goes to 1
    Francesco Della Pietra, Carlo Nitsch, Francescantonio Oliva, Cristina Trombetti

216 Eigenvalue problems for the $p$–Laplacian in the critical range $1 < p < 2$
    José C. Sabina de Lis

217 Existence and uniqueness for the inhomogeneous 1-Laplace evolution equation revisited
    Marta Latorre, Sergio Segura de León
MS08. PROGRESS ON TIME INTEGRATORS FOR ODE

Organized by T. Roldán and J.I. Montijano

219 Analyzing and extending existing classes of methods by means of the theoretical framework of General Linear Methods
Giuseppe Izzo

220 Low order, low storage exponentially fitted explicit Runge-Kutta methods
José María Franco, Inmaculada Higueras, Juan Ignacio Montijano, Luis Rández

221 Highly stable explicit numerical methods with Jacobian dependent coefficients for differential problems
Dajana Conte, Giovanni Pagano, Beatrice Paternoster

222 Parallel-in-time splitting-based methods for the solution of parabolic equations
Andrés Arrarás, Francisco Gaspar, Laura Portero, Carmen Rodrigo

223 Efficient Runge-Kutta-TASE methods for the solution of Stiff problems
Manuel Calvo, Juan I. Montijano, Luis Rández

224 Solving differential equations using neural networks: Pros and Cons
Erick Eduardo Ramírez Torres, Antonio Rafael Selva Castañeda, Juan Ignacio Montijano Torcal

225 Efficient time integration of nonlinear partial differential equations by means of Rosenbrock methods
Isaías Alonso-Mallo, Begoña Cano

226 Boundary corrections for splitting methods on multi-dimensional PDEs
Severiano González-Pinto, Domingo Hernández-Abreu, Soledad Pérez-Rodríguez

227 Taylor-Fourier integrators
Mari Paz Calvo, Joseba Makazaga, Ander Murua

228 A new insight on positivity and contractivity of Crank-Nicolson scheme
Inmaculada Higueras, Teo Roldán

MS09. ORTHOGONAL POLYNOMIALS, SPECIAL FUNCTIONS AND APPROXIMATION THEORY

Organized by José Luis López García, Chelo Ferreira and Ester Pérez Sinusía

230 Orthogonal polynomials on the unit circle and functional differential equations
María José Cantero, A. Iserles
Ladder operators and a second–order difference equation for Sobolev–type orthogonal polynomials

*Galina Filipuk, Juan F. Mañas, Juan J. Moreno–Balcázar*

Sobolev orthogonal polynomials and spectral methods in boundary value problems

*Francisco Marcellán*

On Sobolev orthogonal polynomials on the simplex

*Misael Marriaga*

Quasi–birth and death processes

*Lidia Fernández, Manuel D. de la Iglesia*

Discrete Appell–Dunkl sequences

*Judit Mínguez*

A convergent and asymptotic Laplace method for integrals

*Jose L. Lopez, Pedro Pagola, Pablo Palacios*

Best algebraic bounds for ratios of modified Bessel functions

*Javier Segura*

---

**MS10. Success Stories between Academia and Industry at CITMaga**

*Organized by María Luisa Rapún and Emilio Carrizosa*

Mathematical modeling and numerical simulation in SisAl project, an innovative pilot for silicon production

*Jorge Albella, José Luis Ferrín, Branca García, Dolores Gómez, Luis Pérez, Pilar Salgado*

Optimization of an industrial paper air-drying line using a reduced model derived from CFD

*Elena Martín, Iván Vieitez, Fernando Varas*

Optimal biddings for a renewable energy production plant with a storage system

*David Aller Giráldez, Alfredo Bermúdez de Castro, Ángel Enrique Cano García, Manuel Cremades Buján, Daniel Prol Cambados, Jerónimo Rodríguez*

Computation of resonances in underwater acoustics using Perfectly Matched Layers

*Sara Recondo-Estévez, Andrés Prieto, Luis Hervella-Nieto*

Physically Based Reduced Order Battery Models Including Heterogeneous Degradation for Real-Time Control Applications

*David Aller Giráldez, Mateo Alonso García Cabezón, Alfredo Bermúdez de Castro, David Casasnovas González, Manuel Cremades Buján, Juan Nicolás Aguado, Jerónimo Rodríguez*

Real-time Boundary Heat Flux Estimation in Continuous Casting Molds Using Data Assimilation

*Gianluigi Rozza, Giovanni Stabile, Umberto Emil Morelli, Peregrina Quintela, Patricia Barral*

Modelling, numerical simulation and optimal control problems related to regasification plants

*Alfredo Bermúdez, Gabriel Capeáns, Ángel Manuel González, Jorge Leira, Christian Peláez, Mohsen Shabani, Rocío Vega*
Thermo-electromagnetic-mechanical simulation of an electric upsetting process in automotive industry based on Lagrangian formulations
Marta Benítez, Alfredo Bermúdez, Pedro Fontán, Iván Martínez, Pilar Salgado

MS11. Reduced Order Modeling Applied to Architecture and Engineering

Organized by Enrique Delgado Ávila, Tomás Chacón Rebollo and Macarena Gómez Mármol

Reduced Order models and methods. Applications to transition spaces in buildings
Cristina Caravaca García, Tomás Chacón Rebollo, Soledad Fernández García, Macarena Gómez Mármol, Samuele Rubino

Numerical resolution of problems with dominant convection by stabilized methods based on data
Tomás Chacón Rebollo, Daniel Franco Coronil

Reduced Basis Modeling for LPS pressure VMS-Smagorinsky model
Tomás Chacon Rebollo, Enrique Delgado Ávila, Macarena Gómez Mármol

Residual-based data-driven Variational Multiscale Reduced Order Models
Birgul Koc, Changong Mou, Samuele Rubino, Tomás R. Chacón, Raffaella De Vita, Traian Iliescu

An approach to Reduced Basis Large Eddy Simulation turbulence models based upon Kolmogorov’s equilibrium turbulence theory
Cristina Caravaca García, Tomás Chacón Rebollo, Macarena Gómez Mármol

Model Order Reduction: A geometric approach
Antonio Falcó

Certified Reduced Order Method for Parametrized Allen-Cahn Equation
Mejdi Azaiez, Chacón Tomas, Wu Liang, Xu Chuanju


Organized by Carmelo Clavero and José Luis Gracia

Efficient time integration of semilinear wave problems with time-dependent boundary values using splitting methods
Isaías Alonso-Mallo, Ana Portillo

A high-order Lagrange–Galerkin method for compressible flows
Manuel Colera, Jaime Carpio, Rodolfo Bermejo

Linearly implicit splitting methods for semilinear singularly perturbed convection-diffusion systems
Carmelo Clavero, Juan Carlos Jorge
Serverless computing architecture for data processing and detecting anomalies in the ESA Mars Express MARSIS instrument
David Pacios, José Luis Vázquez-Poletti, Beatriz Sánchez-Cano, Luis Vázquez

A posteriori error estimation for an EG method in fluid mechanics
Vivette Girault, María González Taboada, Frédéric Hecht

Numerical methods for singularly perturbed reaction-diffusion problems with non-smooth data
José Luis Gracia, Eugene O’Riordan

**MS13. EFFICIENT SOLVERS FOR LARGE SPARSE LINEAR SYSTEMS**

Organized by Carmen Rodrigo and Francisco J. Gaspar

Solvers for Multiphysics Problems in Brain Biomechanics
Ana Budiša, Xiaozhe Hu, Miroslav Kuchta, Kent-Andre Mardal, Ludmil Zikatanov

A posteriori error estimates in finite element method by preconditioning
Yuwen Li, Ludmil Zikatanov

Nonlinear solver acceleration based on machine learning applied to multiphase porous media flow
Vinicius L. Santos Silva, Pablo Salinas, Matthew D. Jackson, Christopher C. Pain

Monolithic Multigrid for a Reduced-Quadrature Discretization of Poroelasticity
James Adler, Yunhui He, Xiaozhe Hu, Scott MacLachlan, Peter Ohm

An efficient solver for IGA discretizations of Biot’s equations
Álvaro Pé de la Riva, Francisco J. Gaspar, Carmen Rodrigo

A geometric multigrid solver for the Biot problem on logically rectangular grids
Javier Zaratiegui, Carmen Rodrigo, Andrés Arrarás, Laura Portero

**MS14. NONLINEAR ANALYSIS IN PARTIAL DIFFERENTIAL EQUATIONS**

Organized by Salvador López-Martínez and José Carmona

Elliptic equations with subquadratic growth in the gradient
José Carmona, Salvador López-Martínez, Pedro J. Martínez-Aparicio

Fractional equations with nonlocal “gradient terms”
Boumediene Abdellaoui, Antonio J. Fernández, Tommaso Leonori, Abdelbadie Younes

Qualitative properties on the $p$-Stokes vectorial system
Rafael López-Soriano

Elliptic problems with mixed boundary data for the spectral fractional Laplacian
Eduardo Colorado
MS15. Industrial Mathematics at the Centre de Recerca Matemàtica

Organized by María Luisa Rapún and Emilio Carrizosa

275 Developing mathematical models for industrial adsorption columns
Tim Myers, Abel Valverde, Maria Agüareles, Francesc Font, Alba Cabrera-Codony

276 Modeling of scale-free marked point processes for hazard assessment
Jordi Baró

277 About noise, transients, and scaling laws close to saddle-node bifurcations
Josep Sardanyés

278 In silico assessment of targetted combination therapies: Bivalent chromatin as a case study
Tomás Alarcón

MS16. Partial Differential Equations and Homogenization

Organized by Pedro J. Martínez-Aparicio and Alexis Molino Salas

280 Minimization and maximization of the first eigenvalue for a two-phase material
Juan Casado-Díaz

281 An asymptotic model by homogenization for elastic beams
Juan Casado-Díaz, Manuel Luna-Laynez, Antonio Pallares-Martín

282 Multiple positive solutions for an elliptic Kirchhoff equation
David Arcoya, José Carmona, Pedro J. Martínez-Aparicio

283 On optimal potential problems
Giuseppe Buttazo, Juan Casado-Díaz, Faustino Maestre

284 Un sistema elíptico no lineal en espacios de Sobolev-Orlicz anisótropos: análisis y simulación numérica
Francisco Ortegón Gallego, Hakima Ouyahya, Mohamed Rhoudaf

MS17. Mathematics in Industry and Organizations

Organized by María Luisa Rapún and Emilio Carrizosa

286 Quality classification of recycled plastics
Francesc Aràndiga, Antonio Baeza, Rosa Donat, Sergio López Ureña, Mari Carmen Martí, Pep Mulet, Dionísio Félix Yañez
Data fitting in a portable post-harvest system. Removal of attached ink in plastic
Francesc Aràndiga, Antonio Baeza, Rosa Donat, Mari Carmen Martí, Pep Mulet

Machine Learning techniques for marketing using social login data
J.M. Camacho Rodríguez, R. Naveiro, D. Ríos Insua

Mesoscopic modelling and simulation of espresso extraction
Chaojie Mo, Luciano Navarini, Marco Ellero

Optimal management of a routing protocol in a call center for staff dimensioning
Carlos Gorria, Mikel Lezaun

Numerical simulations for tsunami preparedness risk assessment in Andalusian coast. Indra-Edanya group collaboration
Carlos Sánchez-Linares, Jorge Macías Sánchez, Manuel Jesús Castro Díaz

**MS18. GEOMETRIC FLOWS AND PDEs IN GEOMETRY**

*Organized by Raquel Villacampa, Antonio Otal and Luis Ugarte*

Generalized Ricci Flow
Mario Garcia-Fernandez

SKT structures and the pluriclosed flow on solvmanifolds
Romina M. Arroyo, Ramiro Lafuente, Marina Nicolini

Supersymmetric spinorial flows on three-dimensional Cauchy hypersurfaces
Carlos Shahbazi, Ángel Murcia

The Chern-Ricci flow on Inoue surfaces
Daniele Angella

Linearity of homogeneous solutions to degenerate elliptic equations in dimension three
José A. Gálvez
PLENARY TALKS
Reduced order models for incompressible flows

Authors:
• Julia Novo (julia.novo@uam.es) Universidad Autónoma de Madrid, Spain

Abstract:
We consider proper orthogonal decomposition (POD) methods to approximate the incompressible Navier-Stokes equations. Our aim is to get error bounds with constants independent of inverse powers of the viscosity parameter. This type of error bounds are called robust. In the case of small viscosity coefficients and coarse grids, only robust estimates provide useful information about the behavior of a numerical method on coarse grids. To this end, we compute the snapshots with a full order stabilized method (FOM). We also add stabilization to the POD method. We study a case in which non inf-sup stable elements are used for the FOM and a case in which inf-sup stable elements are used. In the last case to approximate the pressure we use a supremizer pressure recovery method.

We show that in case we have some a priori information about the velocity, a POD data assimilation algorithm converges to the true solution exponentially fast improving the accuracy of the standard POD method.

In practical simulations one can apply some given software to compute the snapshots. It could then be the case that a different discretization for the nonlinear term is used in the FOM and the POD methods. We analyze the influence of using different discretizations for the nonlinear term.

Finally, we also analyze the influence of including snapshots that approach the velocity time derivative. We study the differences between projecting onto $L^2$ and $H^1$ and prove pointwise in time error bounds in both cases.
Numerical simulation of flow problems in deformable porous media

Authors:
- Carmen Rodrigo (carmenr@unizar.es) IUMA, Universidad de Zaragoza

Abstract:
The numerical simulation of coupled mechanical deformation and fluid flow in porous media has become of increasing importance in several branches of technology and natural sciences. Geothermal energy extraction, CO2 storage, hydraulic fracturing or cancer research are among typical societal relevant applications of poromechanics.

In this talk, we will introduce the Biot’s model of poroelasticity, and we will focus on the numerical difficulties that appear in its numerical solution. More concretely, we will treat mathematical and practical aspects of models for poroelasticity, with an emphasis on the analysis of stable numerical discretizations and the efficient solution methods for the coupled system of partial differential equations.

Robust discretizations with respect to all the physical parameters are needed for this type of problems to obtain reliable numerical solutions. This is a very important task, and some efforts are being carried out in this address by the scientific community. Another important aspect in the numerical simulation of poromechanics problems deals with the efficient solution of the large systems of algebraic equations obtained after discretization. This is the most consuming part when real simulations are performed, and for this reason, a lot of effort has been made in the last years to design efficient solution methods for these problems.

In this talk, we address the key points and the recent developments in numerical methods for poromechanics.
Hypocoercivity-preserving Galerkin discretisations

Authors:

- Emmanuil Georgoulis (georgoulis@math.ntua.gr) Univ. Leicester, United Kingdom & NTU Athens, Greece & IACM-FORTH, Greece

Abstract: Degenerate differential evolution PDE problems are often characterised by the explicit presence of diffusion/dissipation in some of the spatial directions only, yet may still admit decay properties to some long time equilibrium. Classical examples include the inhomogeneous Fokker-Planck equation, Boltzmann equation with various collision kernels, systems of equation arising in micromagnetism or flow vorticity modelling, etc. In the celebrated AMS memoir ”Hypocoercivity”, Villani introduced the concept of hypocoercivity to describe a framework able to explain decay to equilibrium in the presence of dissipation in some directions only. The key technical idea involved is to exploit certain commutators to overcome the degeneracy of dissipation.

I shall present some results and ideas on the development of numerical methods which preserve the hypocoercivity property upon discretisation. As a result, such numerical methods will be suitable for arbitrarily long-time simulations of complex phenomena modelled by kinetic-type formulations. This will be achieved by addressing the key challenge of lack of commutativity between differentiation and discretisation in the context of mesh-based Galerkin-type numerical methods via the use of carefully constructed non-conforming weak formulations of the underlying evolution problems.
Dynamics of excitable cells: neurons and cardiomyocytes

Authors:
• Roberto Barrio (rbarrio@unizar.es) IUMA, Universidad de Zaragoza

Abstract: In recent years, much attention has been paid to the description of excitable media, such as the dynamics of the brain and heart. In both cases, the building blocks are excitable cells, neurons, and cardiomyocytes, and a detailed look at the mathematics behind some of their mathematical models provides a good starting point for answering some observed phenomena. In this talk we show how some apparently simple phenomena, such as the spike-adding process, have important consequences in the models and how various elements intervene behind their formation, such as homoclinic bifurcations, fast-slow decompositions, ”canards”, the completion of the Smale topological template, the formation of Morse surfaces creating geometric bifurcations, etc. Finally, we will illustrate its relevance in insect gait patterns and in the formation of cardiac arrhythmias.

This talk is based on joint works with Santiago Ibañez, Álvaro Lozano, M. Ángeles Martínez, Lucía Pérez, Esther Pueyo, Marcos Rodríguez and Sergio Serrano.
From linear to nonlinear \( n \)-width: optimality in reduced modelling

Authors:
- Albert Cohen (albert.cohen@sorbonne-universite.fr) Laboratoire Jacques-Louis Lions, Sorbonne Université, France

Abstract:
The concept of \( n \)-width has been introduced by Kolmogorov as a way of measuring the size of compact sets in terms of their approximability by linear spaces. From a numerical perspective it may be thought as a benchmark for the performance of algorithms based on linear approximation. In recent years this concept has proved to be highly meaningful in the analysis of reduced modeling strategies for complex physical problems described by parametric problems. This lecture will first review significant results in this area that concern (i) the practical construction of optimal spaces by greedy algorithms and (ii) the preservation of the rate of decay of widths under certain holomorphic transformation. It will then focus on recent attempts to propose non-linear version of \( n \)-widths, how these notions relate to metric entropies, and how they could be relevant to practical applications.
Port-Hamiltonian systems: a new paradigm for modeling, simulation and control of complex systems

Authors:
• Volker Mehrmann (mehrmann@math.tu-berlin.de) TU Berlin, Germany

Abstract: Most real world dynamical systems consist of subsystems from different physical domains, modeled by partial-differential equations, ordinary differential equations, algebraic equations, combined with input and output connections. To deal with such complex systems, in recent years the class of dissipative port-Hamiltonian (pH) systems has emerged as a very efficient new modeling methodology. The main reasons are that the network based interconnection of pH systems is again pH, Galerkin projection in PDE discretization and model reduction preserve the pH structure and the physical properties are encoded in the geometric properties of the flow as well as the algebraic properties of the equations. Furthermore, dissipative pH system form a very robust representation under structured perturbations and directly indicate Lyapunov functions for stability analysis. We discuss dissipative pH systems and describe, how many classical models can be formulated in this class. We illustrate some of the nice algebraic properties, including local canonical forms, the formulation of an associated Dirac structure, and the local invariance under space-time dependent diffeomorphisms. The results are illustrated with some real world examples.
Asymptotic preserving schemes for hyperbolic systems of balance laws

Authors:
- Giovanni Russo (giovanni.russo1@unict.it) Department of Mathematics and Computer Science, University of Catania, Italy

Abstract: Several physical systems are described by evolutionary partial differential equations which have the structure of hyperbolic systems of balance laws of the form

\[ U_t + F(U)_x = \frac{1}{\epsilon} R(U), \]

where \( U(x, t) \in \mathbb{R}^M \) and the right hand side may contain a stiff relaxation term as the parameter \( \epsilon \) becomes small. Some specific examples include discrete kinetic models, shallow water with friction, extended thermodynamics.

After finite difference space discretization, one obtains a large system of ODEs. Such systems are efficiently solved by implicit-explicit schemes (IMEX) [1], which treat explicitly the non-stiff hyperbolic term, and implicitly the stiff source term [2]. If \( R \) is a relaxation, this means that i) there exists a matrix \( Q \in \mathbb{R}^{m \times M}, m < M \), such that \( QR(U) = 0 \forall U \in \mathbb{R}^M \) and ii) the solutions of \( R(U) = 0 \) can be uniquely expressed as \( U = E(u) \), with \( u = QU \in \mathbb{R}^m \). As a result, as \( \epsilon \to 0 \) the system formally relaxes to a smaller system of conservation laws of the form

\[ u_t + f(u)_x = 0 \]

with \( f = QF(E(u)) \). This reduction for vanishing \( \epsilon \) can be made rigorous if a suitable subcharacteristic condition is satisfied [3]. Such a property has been adopted to construct the relaxation methods [4]. A scheme for the numerical solution of system (I) which becomes a consistent scheme for system (2) as the relaxation parameter vanishes is said to be Asymptotic Preserving (AP) [2], [5].

The long time behaviour of such systems may be better described by a system containing parabolic terms (hyperbolic to parabolic relaxation). In such cases a different IMEX strategy is needed to capture the correct diffusion limit, and to avoid the parabolic CFL restriction on the time step.

IMEX schemes can be also adopted for the numerical solution of systems in which the stiffness does not appear in the additive form of Eq.(1), thus allowing the construction of high order schemes for non-linear problems which are much more cost-effective than fully implicit schemes.

For moderate values of \( \epsilon \), there might be non-trivial stationary solutions of problem (I). If one is interested in solving problems whose solution is a small perturbation of a stationary one, then it is highly advisable to adopt a Well-Balanced scheme (WB), i.e. a scheme which is able to maintain, within machine precision or to great accuracy, the (discrete or continuous) stationary solution, otherwise truncation error may be comparable or even greater than the small perturbation one is interested in [6].

If the source term contains a stiff relaxation and a non stiff term, i.e. for systems of the form

\[ U_t + F(U)_x = \frac{1}{\epsilon} R(U) + G(U, x), \]

(3)
in the limit of vanishing $\epsilon$ the system relaxes to a lower dimensional system of balance laws of the form

$$u_t + f(u)_x = g(u, x), \quad (4)$$

where $g(u, x) = QG(E(u), x)$. In such cases nontrivial equilibria appear for all values of $\epsilon$. Effective treatment of such problems for small to vanishing values of $\epsilon$ requires schemes which are at the same time AP and WB.

A slightly different class of problems appears when both the hyperbolic term and the source are (equally) stiff:

$$\epsilon U_t + F(U)_x = R(U) \quad (5)$$

In such cases the system may relax to a stationary solution in a very short time. If one is interested in efficiently capturing the stationary solution, then it is advisable to adopt an implicit (or semi-implicit) scheme which is at the same time asymptotic preserving and well-balanced.

The purpose of the talk is to illustrate how to adopt IMEX of semi-implicit strategy to solve various kinds of systems containing stiff and non stiff terms, and how to combine them with modern high-order well-balanced finite volume discretization [7], [8], thus obtaining asymptotic-preserving well-balanced schemes for the classes of problems mentioned above.

Several numerical examples will be presented, which show the effectiveness of the approach.

The research is conducted mainly in collaboration with Sebastiano Boscarino (University of Catania), Lorenzo Pareschi (University of Ferrara) and with Carlos Parés, Manuel Castro, and Irene Gomez-Bueno (University of Malaga).

Acknowledgements
This research has been partially supported by the Italian Ministry of Instruction, University and Research (MIUR) with funds coming from PRIN Project 2017 (No. 2017KKJP4X entitled "Innovative numerical methods for evolutionary partial differential equations and applications").

References:


Optimal control in soft robotics

Authors:
• Francisco Periago (f.periago@upct.es) Universidad de Cartagena, Spain

Abstract: Since the early 1940s, the field of robotics has evidenced a paradigm shift from conventional hard robotics to soft robotics, through the exploration of machines or components with biomimetic dexterous features capable of superseding the ability of humans whilst safely interacting with them. Soft robots are highly nonlinear systems made of highly deformable materials such as elastomers, polymers and other soft matter, that often exhibit intrinsic uncertainty in their elastic responses under large strains due to microstructural inhomogeneity. As a consequence, control of soft robots, potentially actuated by means of a wide spectrum of complex external stimuli (electric or magnetic field, mechanical pressure, osmotic pressure, etc.) is not a trivial task.

This presentation will review on modelling, mathematical analysis, control and design of soft materials. The theoretical analysis and numerical modeling relies on the theory of hyperelasticity. In addition to the constitutive model, another aspect of paramount importance in optimal control of soft materials is the choice of the cost functional to be minimized. Tracking-type cost functionals based on distance functions, such as the Hausdorff distance, have been recently proposed as a natural choice in this field. The numerical treatment of uncertainty quantification is another difficulty due, among others, to the well-known phenomenon of the curse of dimensionality. All of these issues will be illustrated in the talk through several concrete examples. Finally, some mathematical challenges in this emergent field will be described.

The original results included in the presentation have been obtained in collaboration with Jesús Martínez-Frutos (UPCT), Carlos Mora-Corral (UAM), Rogelio Ortigosa-Martínez (UPCT) and Pablo Pedregal (UCM).
TALK AWARD WINNERS
Theory and methods for random differential equations: An overview and some recent results

Authors:
• Marc Jornet (marc.jornet@uv.es) Departament de Matemàtiques, Universitat de València, 46100 Burjassot, Spain

Abstract:
Given a model of differential equations, simulations are needed to predict the response behavior. To achieve this target, apart from having efficient integrators, one should keep in mind the uncertainties associated to the phenomenon and data collection. These provide variability to the input parameters of the model. Thus, it is more realistic to treat the inputs as random variables, which makes the model solution a stochastic process and gives rise to a random differential equation (RDE) [1]. The statistical content of the response (mean, variance, density) is the main interest. The field of uncertainty quantification (UQ) is devoted to the impact of uncertainty on parameters (inverse problem) and on the output (forward problem) [2].

In this communication, I will focus on theory and numerical methods for RDEs. First, I will talk about strong solutions and density representation through Liouville’s equation [3, 4]. Second, I will briefly expose techniques for inverse UQ [5]. And finally, I will give details on methods for forward UQ based on stochastic expansions [6]. Possible research directions on RDEs will be suggested.

References:
Concentration versus Simplification in Aggregation-Diffusion Equations

Authors:
• David Gómez-Castro (gomezcastro@maths.ox.ac.uk) U. of Oxford

Abstract: Over the last two decades, intense work has been devoted to the Aggregation-Diffusion equation
\[ \partial_t \rho = \text{div} \left( \rho \nabla (U'(\rho) + V + W * \rho) \right). \]
This family of problem model, amongst other things, the mean-field limit of systems with a large number of interacting particles arising in biology. Their mathematical structure is rather interesting, since they can be seen as the gradient flow in a suitable metric of a free energy functional in a suitable topology. Thus, we can expect the system to asymptotically approximate minimisers as \( t \to \infty \). This is known to be true when the energy is suitably convex. In the case of Fast Diffusion \( (U(\rho) = \rho^m/(m-1) \) with \( 0 < m < 1 \)), this can fail, so the gradient flow techniques do not guarantee convergence to the minimiser.

There is a long literature characterising the minimisers of the energy functional and, in particular, discussing the existence or not of delta Deltas. The presence of a delta is usually known as a concentration phenomena. On the opposite end of the spectrum there is asymptotic simplification: decay to zero with a given profile (as in the case of the heat equation).

In this talk, I will first review classical known results of formation of Diracs and asymptotic simplification; and then present two recent works. On the one hand, the asymptotic formation of Dirac deltas in the case of Fast Diffusion [1]. On the other, a general result of asymptotic simplification to the heat kernel [2] when \( W \) is bounded, with suitably integrable derivatives.

The talk presents joint work with J.A. Carrillo, J.L. Vázquez, Y. Yao, and C. Zeng.

References:

Some open problems in dynamical systems

Authors:
• Armengol Gasull (gasull@mat.uab.cat) Departament de Matemàtiques (Univ. Autònoma de Barcelona) and Centre de Recerca Matemàtica

Abstract:
In my recent paper [1] I give a list of 33 open problems that I have found along the years in my research. In this talk I will present a selection of some of them. More concretely, I will include some particular cases of Hilbert 16th problem, which recall deals with the number limit cycles of planar polynomial ordinary differential equations; problems related with the behavior of the period function associated to a continuum of periodic orbits; questions about Abel differential equations, piecewise linear differential equations; difference equations; dynamical geometrical problems; problems about global attraction; questions involving polynomials; and finally some questions of recreational mathematics with a dynamical flavor. I will also try to contextualize all of them.

References:
Numerical analysis of some nonlinear hyperbolic systems of Partial Differential Equations arising from Fluid Mechanics

Authors:
• Ernesto Pimentel-García (erpigar@uma.es) University of Málaga

Abstract:
This thesis addresses four different problems related to the numerical analysis of hyperbolic systems of nonlinear partial differential equations. These problems are related to the numerical resolution of mathematical models of fluid mechanics in applications related to shallow water flows and gas dynamics in the context of classical or relativistic mechanics.

The first problem addressed is the study of the Riemann problem for the shallow water equations on a step-shaped bottom, in the particular case in which there is only water on one side of the step: see [1].

The next question to be addressed is the efficient implementation of numerical methods based on approximate Riemann solvers and, in particular, the Roe method, which is based on solving linearized Riemann problems in the intercells: see [2].

The next objective is to make a systematic study of the asymptotic behavior of the solutions of the relativistic Burgers and Euler models based on the Schwarzschild metric, using numerical methods: see [3].

The last problem that arises in the thesis is the extension to second-order of a technique developed to ensure the correct convergence of the numerical solutions in the case of systems with nonconservative products: see [4].

References:
P.1. Partial Differential Equations
Existence of solution for a class of semilinear problem with an unknown Radon measure data

Authors:
• Juan Francisco Padial (jf.padial@upm.es) Universidad Politécnica de Madrid

Abstract: The classical semilinear problems $-\Delta u(x) = F(x,u(x)), \ x \in \Omega \subset \mathbb{R}^N$ (with boundary conditions) has been intensively studied in the last century when $F$ is a given function $F : \Omega \times \mathbb{R} \to \mathbb{R}$. Nevertheless, some models in Physics or Mechanics can be expressed as $-\Delta u(x) = \mu(x,u)$ in $\mathcal{D}'(\Omega)$ where $\mu(x,u)$ is a Radon measure depending on the own solution $u$, unknown a priori. For instance, this type of problems arise in some ideal fluid dynamics or optimal design context (Bernoulli-type problem). This class of elliptic problems were studied in Diaz–Padial–Rakotoson 2007. Our aim is to study the existence of solutions for a nonlinear Bernoulli-type free boundary problem for the evolution case with a unknown measure data asociated to the (interior) Bernoulli problem. We introduce a semi-implicit time differencing in order to obtain a family of elliptic problems like the cases studied in Diaz–Padial–Rakotoson 2007. For each one of this problems, we will apply a general mountain pass principle due to Ghoussoub-Preiss in order to find a weak solution for a sequence of approximate nonsingular problems. Finally, passing to the limit thanks to some a priori estimates, we obtain the solution, first for each elliptic problem that comes that semi-implicit schema, and later, passing to the limit, we obtain a weak solution for the original parabolic problem.

References:
Existence and regularity of solutions in a semilinear problem with singularity in the datum.

Authors:
• José Carmona Tapia (jcarmona@ual.es) Universidad de Almería
• Antonio J. Martínez-Aparicio (ama194@ual.es) Universidad de Almería
• Pedro J. Martínez-Aparicio (pedroj.ma@ual.es) Universidad de Almería
• Miguel Martínez Teruel (mmteruel@go.ugr.es) Universidad de Granada

Abstract: We prove the existence of a solution in $H^1_0(\Omega)$ of the problem

$$
\begin{cases}
-\text{div}(M(x)\nabla u) = \frac{f(x)}{u^\gamma} & \text{in } \Omega, \\
u = 0 & \text{on } \partial \Omega,
\end{cases}
$$

for $\gamma > 1$, where $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ is a bounded open set satisfying the inner sphere condition and $M(x)$ is a measurable, bounded, elliptic matrix.

This problem has been studied in [1] and [2]. However, the existence of a solution in $H^1_0(\Omega)$ is not always guaranteed.

In this work, we impose some hypotheses in order to assure such regularity on the solution. On the datum, we assume that

$$
0 \leq f(x) \in L^1(\Omega).
$$

We also consider that there exists $m_1, m_2 > 0$ and an open environment in $\Omega$ of $\partial \Omega$, denoted by $\Omega_\delta$, in which

$$
m_1 \varphi_1^\delta \leq f(x) \leq m_2 \varphi_1^\delta \text{ a.e. in } \Omega_\delta,
$$

where $\varphi_1$ is the eigenfunction associated to the first eigenvalue $\lambda_1$ of the operator $-\text{div}(M(x) \cdot \cdot)$ on $\Omega$ with Dirichlet boundary conditions. In $\Omega \setminus \Omega_\delta$, we assume that $f(x)$ is greater than a positive constant, i.e., that there exists $c_0 > 0$ such that

$$
f(x) \geq c_0 > 0 \text{ a.e. in } \Omega \setminus \Omega_\delta.
$$

References:

Convergence of weak solutions of semilinear elliptic problems with datum in $L^1$

Authors:
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- Miguel Martínez-Teruel (mmteruel@go.ugr.es) Universidad de Granada

Abstract:

In [1], the problem

\[
\begin{cases}
- \text{div}(M(x)\nabla u) + a(x)g(u) = f(x) & \text{in } \Omega, \\
u = 0 & \text{on } \partial\Omega,
\end{cases}
\]

\[\text{ (P)}\]

is studied, where $\Omega \subset \mathbb{R}^N$ ($N \geq 2$) is a bounded open set, $M(x)$ is a bounded elliptic matrix and $a(x), f(x) \in L^1(\Omega)$ with $a(x) \geq 0$. In that paper, it is shown that if $g: \mathbb{R} \to \mathbb{R}$ is continuous, strictly increasing and odd with $\lim_{s \to +\infty} g(s) = +\infty$ and there exists a constant $Q > 0$ such that

\[f(x) \leq Qa(x) \text{ a.e. in } \Omega,\]

then there exists a weak solution $u \in H^1_0(\Omega) \cap L^\infty(\Omega)$ of (P) satisfying that $\|u\|_\infty \leq g^{-1}(Q)$.

In this work, we use the concept of entropy solution introduced in [2] to answer a question that is closely related to this result: if we take $a(x) = \frac{1}{n}|f(x)|$ in (P), what is the behavior of $\{u_n\}$, the sequence of weak solutions of (P)?

References:


A nonlocal model of Hyperelasticity and vector calculus for (one-point) nonlocal gradients over bounded domains

Authors:

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• Javier Cueto (Javier.Cueto@uclm.es) UCLM
• Carlos Mora-Corral (carlos.mora@uam.es) UAM

Abstract: This talk is composed of final results from [4] ([3]) and some recent follow-up ones. In particular, after some previous studies regarding bond-based peridynamics and fractional gradient functional, some relevant tools such as nonlocal versions of the Fundamental Theorem of Calculus and Poincaré-Sobolev inequalities are obtained for nonlocal gradients defined for $u : \Omega \cup B(0, \delta) \rightarrow \mathbb{R}$ as

$$D^\delta u(x) = c_{n,s} \int_{B(x,\delta)} \frac{u(x) - u(y)}{|x - y|} \frac{x - y}{|x - y|} \frac{w_\delta(x - y)}{|x - y|^{n-1+s}} dy, \quad x \in \Omega.$$ 

This means we would be dealing with a more realistic framework for applications, in particular Solid Mechanics than the (pure) fractional one [1], since the operators in the latter are defined over the whole space. Both cases were considered as an alternative to nonlinear bond-based peridynamic energies (double integrals) since in such case very few hyperelastic functions could be recovered after the localization process [2]. This analysis is used in order to tackle a nonlocal model of Hyperelasticity under nonlocal Dirichlet conditions over a collar of width $2\delta$. Additionally, some recent results and current research lines in collaboration other authors may include further nonlocal vector calculus, Helmholtz decompositions or quasi-convexity as a characterization of weak lower semi-continuity for these operators.

References:


Weak boundary conditions for Boussinesq flow with free outflow, Part I: Analysis

Authors:
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• Dandy Rueda Castillo (rueda-castillo@math.uni-kiel.de) Universidad Nacional Agraria La Molina, Peru

Abstract: We propose boundary conditions for incompressible flows to model free outflow. In comparison with the standard do-nothing condition, we add certain nonlinear terms to the boundary conditions. We present the method for the Navier-Stokes and the Boussinesq system. In the latter case of non-constant temperature, the boundary condition for the temperature should be modified as well. For stability reasons, we consider an implementations of all Dirichlet boundary conditions by Nitsche’s method. The resulting system is stable and allows for proving existence of weak solutions in 2D and 3D. In order to obtain a stable discrete system we use equal-order finite elements with local projection stabilization. We give an error estimate for the discrete solution.

References:

Degenerate Kirchhoff-type elliptic equations with convective reaction

Authors:
• Calogero Vetro (calogero.vetro@unipa.it) University of Palermo, Department of Mathematics and Computer Science, Via Archirafi 34, 90123, Palermo, Italy

Abstract: Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with smooth boundary $\partial \Omega$, and let $W^{1,p(x)}_0(\Omega)$ denote the anisotropic variable exponent Sobolev space. We consider non-linear elliptic equations involving degenerate Kirchhoff-type operators based on the $p(x)$-Laplacian $\Delta_{p(x)} u = \text{div}(|\nabla u|^{p(x)-2}\nabla u)$ for all $u \in W^{1,p(x)}_0(\Omega)$. We study the equation under Dirichlet, Navier and no-flux boundary conditions. Moreover, the reaction term of the equation is a sufficiently regular function with gradient dependence (convection), as we are interested in covering the physical situations where convective phenomena of fluid dynamics cannot be neglected. On this basis, the Galerkin’s approach together with appropriate a priori estimates, is involved to establish the existence of solutions in a weak sense. The precise strategy is an approximating process involving a suitable Nemitsky map linked to the reaction term, and a discretization of the framework space constructing its Galerkin basis. We conclude the result using some convergence and fixed-point arguments. We design technical conditions to control the growth of the reaction term, obtaining the above mentioned a priori estimates and certain sign constraints.

Keywords: Convection; Galerkin approximation; Kirchhoff type term.

2020 Mathematics Subject Classification: 35J60; 65N30.

References:
Fractional Laplacian BVP with nonlinearities having multiple zeroes

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- Rubén Fiñana Aránego (rubenfa7@hotmail.com) Universidad de Almería
- José Carmona Tapia (jcarmona@ual.es) Universidad de Almería

Abstract:
We study the existence and the multiplicity of solutions to the following nonlinear elliptic problem involving the fractional Laplacian:

\[
\begin{align*}
(-\Delta)^s u &= \lambda f(u) \quad \text{in } \Omega, \\
u &= 0 \quad \text{in } \mathbb{R}^N \setminus \Omega,
\end{align*}
\]

(1)

where \( s \in (0,1) \), \( \Omega \subset \mathbb{R}^N \) (\( N \geq \max\{2, 3 - 2s\} \)), is a bounded domain with smooth boundary, \( \lambda \) is a nonnegative constant and \( f \) is a nonlinear function under some certain assumptions, with a finite family of zeroes, \( a_1, \ldots, a_m \).

The main contributions are related those in [2] and [1] for the local case. More precisely we prove:

1. A necessary condition for the existence of a solution of the problem (1), in terms of the area enclosed by \( f \) between its zeros.
2. Multiplicity of solutions to (1) for large \( \lambda \).

References:


Concentration phenomena in the Lifshitz–Slyozov system with inflow boundary conditions

Authors:
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• Erwan Hingant (erwan.hingant@u-picardie.fr) Université de Picardie Jules Verne & Cogitamus Lab
• Romain Yvinec (romain.yvinec@inrae.fr) INRAE & Université de Tours & Cogitamus Lab

Abstract:
The Lifshitz-Slyozov system describes the temporal evolution of a mixture of monomers and aggregates, where monomers can attach to or detach from already existing clusters. The aggregate distribution solves a transport equation with respect to a size variable, whose transport rates are coupled to the dynamic of monomers in a nonlocal fashion. Recent applications to protein polymerization processes introduce attachment and detachment rates that require a nonlinear boundary condition at zero size. In this contribution we study the long time behavior for different sets of such reaction rates [1, 2], paying particular attention to the onset of concentration phenomena. Since those are not observed experimentally, we discuss to what extent is this concentration behavior generic and we present some modifications of the model that avoid it.

References:
Stieltjes Parabolic Partial Differential Equations. Application to a Digital Twin for a population dynamics problem

Authors:

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- Juan J. Nieto Roig (juanjose.nieto.roig@usc.es) CITMAga, Universidade de Santiago de Compostela, Spain
- F. Adrián F. Tojo (fernandoadrian.fernandez@usc.es) CITMAga, Universidade de Santiago de Compostela, Spain

Abstract:

Stieltjes differential equations, which contain equations with impulses and equations on time scales as particular cases, consist on replacing usual time derivatives by time derivatives with respect to a nondecreasing function. Stieltjes differential equations are suitable to study populations which exhibit dormant states and/or very short (impulsive) periods of reproduction [3]. Recently the authors have also shown in [1] that Stieltjes differential equations can be used to study, from a mathematical point of view, the concept of Digital Twin.

In this work we introduce the concept of Stieltjes Parabolic Partial Differential Equation [2]. We present a mathematical definition of the solution and we also analyze the existence and uniqueness of solution. The advantage of considering the spatial variable in the mathematical model allows us to study situations in which ordinary differential equations are not adequate, such as population dynamics problems where the spatial distribution is relevant.

Finally we will present an application to a Digital Twin based on a Stieltjes Parabolic Partial Differential Equation for a population dynamics problem related to the evolution of the Asian hornet in Galicia.

References:

A general spatial predator-prey model

Authors:
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• Eduardo Muñoz-Hernández (eduardmu@ucm.es) UCM

Abstract:
This communication analyzes the existence and the uniqueness of coexistence states for the generalized spatially heterogeneous predator-prey model

\[
\begin{align*}
\mathcal{L}_1 u &= \lambda u - a(x)u^2 - b(x)\frac{uv}{1 + m(x)u} \quad \text{in } \Omega, \\
\mathcal{L}_2 v &= \mu v + c(x)\frac{uv}{1 + m(x)u} - d(x)v^2 \quad \text{in } \Omega, \\
\mathcal{B}_1 u = \mathcal{B}_2 v &= 0 \quad \text{on } \partial \Omega,
\end{align*}
\]

(1)

where \( \Omega \) is a bounded domain of \( \mathbb{R}^N \) whose boundary, \( \partial \Omega \), is a \( N - 1 \) dimensional manifold of class \( C^2 \), \( \mathcal{L}_1 \) and \( \mathcal{L}_2 \) are second order uniformly elliptic operators, and \( \mathcal{B}_1 \) and \( \mathcal{B}_2 \) are general boundary operators of mixed type.

In this model, \( a > 0, \; d > 0, \; b \geq 0, \; c \geq 0 \) and \( m \geq 0 \).

In (1), \( m(x) \) measures the level of saturation of the predator at any particular location \( x \in \Omega \) where \( m(x) > 0 \), while saturation effects do not play any role if \( m(x) = 0 \). Thus, (1) combines, within the same territory, the classical interactions of Lotka–Volterra type in the region \( m^{-1}(0) \) with the Holling-Tanner functional responses in \( \{ x \in \Omega : m(x) > 0 \} \). So, integrating at a single prototype model the classical Lotka–Volterra and Holling–Tanner kinetics. In this communication, \( \lambda > 0 \) and \( \mu \in \mathbb{R} \) are regarded as bifurcation parameters. In applications, \( \lambda - c^1(x) \) and \( \mu - c^2(x) \) stand for the growth rates of the prey and the predator in the absence of each other.

References:
Lie symmetries and exact solutions for a fourth-order nonlinear diffusion equation

Authors:
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- Tamara María Garrido (tamara.garrido@uca.es) University of Cadiz
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- Rafael de la Rosa (rafael.delarosa@uca.es) University of Cadiz

Abstract:
In this work, we consider a fourth-order nonlinear diffusion partial differential equation, depending on two arbitrary functions. Firstly, we perform an analysis of the Lie symmetries for this parabolic partial differential equation. The analysis is based on the invariance property of the symmetry transformation groups, that yields the infinitesimal generators. By using this invariance condition, we present a complete classification of the Lie point symmetries for the different forms of the functions considered in the equation. Afterwards, the optimal systems of one-dimensional subalgebras for each maximal Lie algebra are determined, by computing previously the commutation relations, with the Lie bracket operator, and the adjoint representation. Next, the symmetry reductions to ordinary differential equations are derived from the optimal systems of subalgebras. Furthermore, we study travelling wave reductions depending on the two arbitrary functions. Finally, some travelling wave solutions are obtained, such as solitons, kinks, and periodic waves.

References:
Predictive ROM based on a coordinate transform technique applied to 1D and 2D transport problems

Authors:

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- José Luis Gracia (jlgracia@unizar.es) IUMA and Department of Applied Mathematics, University of Zaragoza
- Adrián Navas-Montilla (anavas@unizar.es) Fluid Dynamic Technology, I3A, University of Zaragoza
- Pilar García-Navarro (pigar@unizar.es) Fluid Dynamic Technology, I3A, University of Zaragoza

Abstract:

Reduced order models (ROMs) based on the proper orthogonal decomposition [3] are frequently used to solve partial differential equations due to their high computational efficiency when compared to full-order models (FOMs). The ROM must be trained via the snapshot method [3] from the results of the FOM computed off-line up to the training time. However, standard ROMs are not able to compute on-line solutions beyond the training time in transport problems. Therefore, a novel strategy has been developed to extrapolate solutions in time [4]. This method is based on a coordinate transformation [1]. It has been applied to linear and non-linear 1D problems and also to 2D linear problems using the Radon transform [2].

References:

A priori estimates for a 1–Laplace evolution equation

Authors:
• Marta Latorre Balado (marta.latorre@urjc.es) Universidad Rey Juan Carlos
• Sergio Segura de León (sergio.segura@uv.es) Universitat de València

Abstract: In this work we deal with a non-homogeneous parabolic Dirichlet problem involving the 1-Laplacian operator. So far, the problem is solved in the homogeneous version if the initial datum belongs to $L^1(\Omega)$. Furthermore, the non-homogeneous case with square integrable initial datum is already studied whenever the source term belongs to $L^1(0,T;L^2(\Omega))$. Our aim is to prove the existence of solution when the source term and the initial datum are merely integrable functions and for this purpose we have obtained a priori estimates. This is a work in progress with S. Segura de León.
P2. Dynamical systems - Ordinary Differential Equations
Traveling waves for Non-degenerated Reaction-Diffusion equations with a possibly bounded flux

Authors:
• Juan Campos (campos@ugr.es) Universidad de Granada

Abstract:
We analyze an extensive class of Fisher-type reaction-diffusion equations with flows in divergence form. We work with regular flows, which may not meet the standard elliptical conditions, but without other types of singularities.

We show that the range of speeds at which classical traveling waves move is an interval unbounded to the right. Contrary to classical examples, the infimum may not be reached. When the flow is elliptic or over-elliptic the minimum speed of propagation is achieved.

The classical traveling wave speed threshold is complemented by another value by analyzing an extension of the first order boundary value problem to which the classical case is reduced. This singular minimum speed can be justified as a viscous limit of classical minimal speeds in elliptic or over-elliptic flows.

We construct a singular profile for each speed between the minimum singular speed and the speeds at which classical traveling waves move. Under additional assumptions, the constructed profile can be justified as that of a traveling wave of the starting equation in the framework of bounded variation functions.

We also show that saturated fronts verifying the Rankine-Hugoniot condition can appear for strictly lower speeds even in the framework of bounded variation functions.
Hopf Bifurcation for a Functional Differential Equation (FDE) with respect to the delay

Authors:
• Alfonso C. Casal (alfonso.casal@upm.es) Universidad Politécnica de Madrid
• Juan Francisco Padial (jf.padial@upm.es) Universidad Politécnica de Madrid

Abstract: In this work, we present some features of solutions of certain Delay or Functional Differential Equations (FDE). In particular, our interest is focused in the analysis of its oscillatory behavior. In some cases, periodic solutions may arise in FDE due to a state dependent delay. In some other cases, with constant delays, they arise from constant or stationary solutions when these undergo a change of its structure as some of the parameters of the equation vary. One of the ways for this is what is called a Hopf bifurcation (for FDE). Motivated by interesting applications, as some models of traffic flow, among others, we consider the case in which the varying parameter are precisely the delays. For both mathematical reasons and better explanations of real phenomena, we present a new non-linear car-following model with reaction delays, from several sources, as drivers and cars. In particular we try to give a starting point of explanation of a peculiar phenomenon as the emergence of a backpropagating congestion wave in motorway traffic, the phenomenon of stop-and-go waves, empirically studied by many authors. The model can be written as an ordinary nonlinear delay differential equation. Its equilibrium solutions correspond to steady traffic. The variation of the mentioned reaction delays leads to instabilities of equilibria and changes of the structure of the solutions. With a number of simulations for values of the delays we can show, semi-numerically, that the corresponding above mentioned change of structure (representing regimes of real traffic) fulfill the conditions for the appearance of a Hopf bifurcation.

References:
Asymptotic behavior of solutions to lattice diffusion equations

Authors:
- Antonín Slavík (slavik@karlin.mff.cuni.cz) Charles University

Abstract: The goal of this talk is to provide an overview of some recent results on the asymptotic behavior of solutions to various types of diffusion (heat) equations on discrete spatial domains, namely:

- The one-dimensional diffusion equation with discrete space and continuous time studied in [1].
- The multidimensional diffusion equation with discrete space and continuous time studied in [2].
- The one-dimensional diffusion equation with discrete space and discrete time studied in [3].

In particular, we focus on initial-value problems with bounded initial data, and provide sufficient conditions for the existence of pointwise and uniform limits of solutions.

References:

Matrix versions of higher-order recurrence relations for Sobolev-type orthogonal polynomials

Authors:

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• Alberto Lastra (alberto.lastra@uah.es) Departamento de Física y Matemáticas, Universidad de Alcalá, Spain

• Francisco Marcellán (pacomarc@ing.uc3m.es) Departamento de Matemáticas, Universidad Carlos III de Madrid, Spain

Abstract: Sobolev-type orthogonal polynomials on the real line satisfy higher-order recurrence relations that can be expressed as (2N+1)-banded symmetric semi-infinite matrices. In this talk, we present several connections between the aforementioned matrices, and the Jacobi matrices associated with the three-term recurrence relation satisfied by the standard sequence of orthonormal polynomials with respect to the 2-iterated Christoffel transformation of a positive Borel measure.

References:

D-concave nonautonomous scalar bifurcation theory and applications in critical transitions

Authors:
• Jesús Dueñas (jesus.duenas@uva.es) Universidad de Valladolid
• Carmen Núñez (carmen.nunez@uva.es) Universidad de Valladolid
• Rafael Obaya (rafael.obaya@uva.es) Universidad de Valladolid

Abstract: In this talk, we study some one-parametric bifurcation problems of dissipative nonautonomous scalar ordinary differential equations with concave derivative (d-concave). Saddle-node, transcritical and pitchfork bifurcation points of minimal sets are found in the skewproduct formalism for this kind of differential equations. Critical transitions are sudden and abrupt changes in the state of a complex system which occur on account of small variations on external parameters. The described bifurcations provide adequate critical transitions models in complex systems.

References:
Lie algebra of linear colored network dynamical systems

Authors:

- Fahimeh Mokhtari (f.mokhtari@vu.nl) Department of Mathematics, Vrije Universiteit Amsterdam
- Jan Sanders (jan.sanders.a@gmail.com) Department of Mathematics, Vrije Universiteit Amsterdam

Abstract:

In [1], we show that for $N$-dimensional vector fields with $C$ colors, different functions describing different types of cells in the network, this Lie algebra $\mathfrak{net}_{C,N}$ is isomorphic to the semidirect sum of a semisimple part, consisting of two simple components $\mathfrak{sl}_C$ and $\mathfrak{sl}_{N-C}$, which we write as a block-matrix and a solvable part, consisting of two elements representing the identity $\mathfrak{C}$ in $\mathfrak{c} \simeq \mathfrak{gl}_C$ and $\mathfrak{B}$ in $\mathfrak{b} \simeq \mathfrak{gl}_B$, and an abelian algebra $\mathfrak{a} \simeq \mathfrak{Gr}(C,N)$, the Grassmannian, consisting of the $C$-dimensional subspaces of $\mathbb{R}^N$. In this talk, we describe this algorithm that gives us the Levi decomposition of colored networks.

References:

Control properties of multiagent singular systems under a geometrical point of view

Authors:
- M. Isabel García-Planas (maria.isabel.garcia@upc.edu) UPC

Abstract: In this work, the controllability of multi-agent singular linear systems that are interconnected through communication channels is analyzed. Multiagent systems are of interest because they have great applicability in multiple areas, such as electrical networks, bioinformatics, sensor networks, vehicles, robotics, and neuroscience. Controllability of a dynamical system has being largely studied by several authors and under many different points of view. Nevertheless, much less effort has been devoted to studying the controllability of multi-agent linear systems taking into account the dynamics taking place on them. In this work, the controllability condition is analyzed through invariant subspaces under feedback and in the case of multi-agent linear systems that can be described by $k$ agents with dynamics $E_i \dot{x}_i = A_i x_i + B_i u_i$, $i = 1, \ldots, k$

References:

Autonomous and non-autonomous unbounded attractors in evolutionary problems

Authors:
• Kalita Piotr (piotr.kalita@ii.uj.edu.pl) Jagiellonian University (Poland)
• García Fuentes Juan (jgfuentes@us.es) Universidad de Sevilla
• Banaśkiewicz Jakub (jakub.banaskiewicz@doctoral.uj.edu.pl) Jagiellonian University

Abstract: Slowly non-dissipative problems in mathematical physics are such models, where the solution does not blow-up in finite time, but, in appropriate sense, can tend to infinity as time goes to infinity. Still, it is possible to define the concept of attractor for such problems - this is the so called unbounded attractor. Such attractor is invariant, it is the countable sum of increasing compact sets, and attracts, in appropriate sense, all bounded sets. The concept of unbounded attractor was first proposed by Chepyzhov and Goritskii [1]. In this work we develop their theory to give abstract conditions which guarantee the unbounded attractor existence. We also discuss their structure and intrinsic properties. Moreover, following the concepts first introduced in [2] we develop the abstract non-autonomous theory of unbounded attractors. Finally, we apply our theory to a parabolic evolutionary PDE: we present new condition which guarantees that the thickness of multivalued inertial manifold related with unbounded attractor decays to zero as its projection on the finite dimensional unstable space tends to infinity, and we use the inertial manifold theory to provide the conditions (smallness of the Lipschitz constant of the nonlinearity) which guarantee that the unbounded attractor is a manifold which exponentially attracts all bounded sets.

References:


About the structure of attractors for a nonlocal Chafee-Infante problem

Authors:
• Rubén Caballero (ruben.caballero@umh.es) Universidad Miguel Hernández
• Alexandre Carvalho (andcarva@icmc.usp.br) Universidade de São Paulo
• Pedro Marín-Rubio (pmr@us.es) Universidad de Sevilla
• José Valero (jvalero@umh.es) Universidad Miguel Hernández

Abstract:
If we consider the non-local equation
\[
\frac{\partial u}{\partial t} - a(u)\frac{\partial^2 u}{\partial x^2} = \lambda f(u)
\]
(1)
with Dirichlet boundary conditions, then it is possible to define a suitable Lyapunov functional. In [1] it is shown that regular and strong solutions generate (possibly) multivalued semiflows having a global attractor which is described by the unstable set of the stationary points. In the case where the function \( f \) is odd and equation (1) generates a continuous semigroup the existence of fixed points of the type given in the Chafee-Infante problem was established in [2]. Moreover, if \( a \) is non-decreasing, then they coincide with the ones in the Chafee-Infante problem. In this work we extend these results for a more general function \( f \). Our goal is to describe the structure of the attractor as accurately as possible. For this aim, we focus on the particular situation where the domain is one-dimensional and the function \( f \) is of the type of the standard Chafee-Infante problem, for which the dynamics inside the attractor has been completely understood [3].

References:
Mutlichromatic Travelling Waves for Lattice Nagumo Reaction Diffusion Equation

Authors:
- Hermen Jan Hupkes (hhupkes@math.leidenuniv.nl) University of Leiden
- Leonardo Morelli (leonardo.morelli@gmail.com) University of Leiden
- Petr Stehlík (pstehlik@kma.zcu.cz) University of West Bohemia
- Vladimír Švígler (sviglerv@kma.zcu.cz) University of West Bohemia

Abstract:
In this talk we discuss multichromatic front solutions to the bistable Nagumo lattice differential equation. Such fronts connect the stable spatially homogeneous equilibria with spatially heterogeneous $n$-periodic equilibria and hence are not monotonic like the standard monochromatic fronts. These multichromatic fronts can disappear and reappear as the diffusion coefficient is increased. In addition, these multichromatic waves can travel in parameter regimes where the monochromatic fronts are also free to travel. This leads to intricate collision processes where an incoming multichromatic wave can reverse its direction and turn into a monochromatic wave.

References:
Large Scale Optimization using Multiresolution Techniques

Authors:
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Abstract: Optimization problems of the type

$$\text{Find } z_{\text{min}} \in \mathbb{R}^N \text{ such that } F(z_{\text{min}}) = \min_{z \in \mathbb{R}^N} F(z), \quad N >> 1, \quad (1)$$

arise frequently in applications, often in connection with the context of calculus of variations, PDE constrained optimization, optimal control or image processing.

In many occasions, specially in engineering computations, it is known, a priori, that the problem has a unique solution which can be computed (from a given initial data) with an appropriate ‘off-the shelve’ (black-box type) optimization technique which, however, tends to be very time consuming. Assuming that problem (1) can be solved by using a given optimization technique, $\mathcal{D}$, we describe a strategy to reduce the computational time required by the direct application of $\mathcal{D}$ on a space with $N$ degrees of freedom. Our technique is based on embedding problem (1) in Harten’s Multiresolution Framework (MRF henceforth) [2], which can be loosely described as a set of mathematical tools to obtain multilevel representations of discrete data sets, similar to those obtained in the well known wavelet framework.

The aim of this talk is to provide a complete mathematical description of the technique used in [1] to solve large-scale optimization problems of the type (1), together with some theoretical results on its properties and performance, including several numerical experiments that confirm our theoretical analysis.

References:


Oscillation-based Numerical Integrators for HMC

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Abstract: Hybrid Monte Carlo (HMC) is a Markov Chain Monte Carlo (MCMC) technique which numerically integrates Hamiltonian dynamics with Hamiltonian \( H = T(p) + U(q) \) to generate proposal samples from a desired probability distribution, \( \exp(-U(q)) \). \( T(p) \) may be freely chosen.

The all-purpose standard for the numerical integration remains the Verlet integrator, which composes the exact flows of the sub-Hamiltonians \( T(p) \) and \( U(q) \). However in many cases, distributions have a Radon-Nikodym density with respect to a Gaussian distribution \([1]\), or may be numerically approximated by a Gaussian multiplied by a residual perturbation \([3]\). In either case, the corresponding Hamiltonian then takes the form of an additive perturbation to a harmonic oscillator. Using ideas from \([2]\), we study integrators which compose the exact flows of \( H_0 = \frac{1}{2}(p^T p + q^T q) \) and \( U_1(q) \) for the Hamiltonian \( H(q,p) = H_0(q,p) + U_1(q) \), with some results on their stability applied to the model problem:

\[
H(q,p) = \frac{1}{2}(p^2 + q^2) + \epsilon q^2.
\]

The integrators are applied to the problem of sampling from the conditioned Stochastic Differential Equation from \([1]\):

\[
dq = -V'(q)d\tau + \sigma dW, \quad q(0) = q(T) = 0
\]

whose distribution \( G \) has a density with respect to the Gaussian \( G_0(q) = \mathcal{N}(0, \sigma^2(-\frac{d^2}{d\tau^2})^{-1}) \) given by \( \frac{dG}{dG_0} \propto \exp\left(-\frac{1}{2\sigma^2} \int_{0}^{T} |V'(q(\tau))|^2 - \sigma^2V''(q(\tau))d\tau \right) \) The spectrum in the analytically tractable case of the Ornstein-Uhlenbeck bridge, with \( V(q) = \frac{1}{2}q^2 \) is used to illustrate the problem.

References:


Reduced dimensionality model for sprint kayak propulsion: applications.

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Abstract: Flatwater canoeing is a human powered displacement sport. If we assume that the main dimension in play is that of displacement and that propulsion can be modeled in terms of a function of time \( f = f(t) \), the equation governing the dynamics is the Newton’s second law, written as:

\[
 m \ddot{v} + d \cdot v^2 = f(t)
\]

Where \( m \) is the mass and \( d \) the drag coefficient. A realistic propulsion could be defined as a piecewise function with aquatic and aerial phases:

\[
f(t) = \begin{cases} 
 at^2 + bt + c & t \in (0, t_w) \\
 0 & t \in (t_w, t_w + t_a)
\end{cases}
\] (1)

And modelling it using Fourier series, the differential equation separates in asymptotic and periodic contributions with the following solution:

\[
v(t) = \sqrt{\frac{f'}{d}} \tanh \left( \sqrt{\frac{d}{f'}} \left( \frac{f'}{m} t + K' \right) \right) + \sum_{n=1}^{6} \left( a_n \cdot \sin(nwt) + b_n \cdot \cos(nwt) \right)
\] (2)

In terms of several parameters. In the present work we analyze this reduced dimensionality model in general and the quadratic stroke approach in particular. We use these approximations first to reproduce field data of this sport and second to extract magnitudes and functional relations from it, for instance, the known quasi-linear relation between stroke frequency and average velocity or the amplitude of velocity oscillations during a displacement with periodic propulsion.
Chaos in the Two-Coupled Brusselators Model: Building a Bridge in the Literature

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Abstract:

The Brusselator is a theoretical model that represents an autocatalytic chemical reaction with oscillations (Belousov-Zhabotinsky reaction). It was introduced in 1968 by I. Prigogine and R. Lefever. If we couple two identical Brusselators by diffusion, we obtain the system that concerns us: the two-coupled Brusselators model. This system has four variables and four parameters (two of them correspond to the coupling).

This coupled system presents different dynamical regimes. For example, in the literature two chaotic regions [1, 2] have been located in the parameter space. The first parametric region with chaotic behaviour [1] is much larger than the other one [2], which has been obtained by applying the Shil’nikov homoclinic theory. The main questions are the following: Are these chaotic regions connected? Which dynamical mechanisms create them? We study in detail the global parameter phase space and how all the different elements are connected. In particular, we use several numerical and analytical techniques, such as spike-counting sweeping, Lyapunov exponents, continuation methods, analytical expansions, Shil’nikov theory, and so on.

References:


Structure of Non-Autonomous Attractors for a Class of Diffusively Coupled ODE

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Abstract: In this poster, we will study the structure of the skew-product attractor for a planar diffusively coupled ordinary differential equation, given by \( \dot{x} = k(y - x) + x - \beta(t)x^3 \) and \( \dot{y} = k(x - y) + y - \beta(t)y^3, \ t \geq 0 \). We identify the non-autonomous structures that completely describes the dynamics of this model giving a Morse decomposition for the skew-product attractor. We also prove the stability under perturbation for the attractor.

References:

[1] A. N. Carvalho, L. R. Rocha, J. A. Langa and R. Obaya, Structure of Non-Autonomous Attractors for a Class of Diffusively Coupled ODE, Discrete and Continuous Dynamical Systems - Series B. Accepted for publication.
Runge-Kutta-Nyström methods for the numerical solution of second order linear inhomogeneous IVPs

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Abstract:
We consider IVPs for $d$-dimensional differential systems of second-order linear inhomogeneous equations given by the equations

$$
y''(t) = f(t, y) = D y(t) + g(t), \quad t \in [t_0, t_0 + T]
y(t_0) = y_0, \quad y'(t_0) = y'_0 \in \mathbb{R}^d,
$$

where $D \in \mathbb{R}^{d \times d}$ is a constant matrix and $g : \mathbb{R} \to \mathbb{R}^d$ is a sufficiently smooth function in the interval of interest.

We approximate the solution of (1) at $t = t_0 + h$ by means of an $s$-stage explicit Runge–Kutta-Nyström (RKN) [1] method given by

$$
y_1 = y_0 + h y'_0 + h^2 \sum_{i=1}^{s} b_i^* G_i, \quad y'_1 = y'_0 + h \sum_{i=1}^{s} b_i G_i,
$$

where the stages $G_i \in \mathbb{R}^d, i = 1, \ldots, s$ are defined by

$$
G_i = f \left( t_0 + c_i h, y_0 + h c_j y'_0 + h^2 \sum_{j=1}^{s} a_{ij} K_j \right), \quad i = 1, \ldots, s
$$

where $b_j^*, b_j, c_j, a_{jk}$ are the coefficients that define the method.

We will derive the order conditions of $s$-stages explicit RKN methods and we will give several $s$-stages RKN methods with order $p = s + 1$.

References:
Chaos Detection: from Lyapunov Exponents to Deep Learning

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Abstract:
One of the main topics in the area of Dynamical Systems is chaos detection. Lyapunov Exponents (LEs) [1] are usually the tool used to decide if for a given initial condition and some values of the parameters, the behaviour of the dynamical system is chaotic or not [2]. However, recently some authors have proposed to solve such problem using Deep Learning (DL) [3], which is the branch of Machine Learning (ML) that uses architectures based on multiple layers (Artificial Neural Networks, ANNs) to learn from data with several levels of abstraction.

In this poster we will explain the state-of-the-art DL techniques that we have trained with time series from some dynamical systems to solve the problem of chaos detection. Moreover, we will analyze how the different architectures perform the task, we will reconstruct the parametric representations of the chaotic dynamics of the systems using the results obtained from the networks and we will compare them to the ones that we can obtain with LEs [2].

References:
High order PPH schemes over nonuniform grids

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Abstract: This work is intended to the construction and implementation over nonuniform grids of a generalization of the family of nonlinear reconstruction operators defined in [1] for uniform grids, and somehow these operators also generalize the results presented in [3]. They are built by using centered piecewise nonlinear polynomial interpolations. The purpose is to obtain high order approximation operators adapted to discontinuities. We define and analyze both theoretically and numerically these operators, and we also work with their associated subdivision schemes.

References:

Generalized Weighted Power Means. Properties and uses.

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Abstract: Our purpose is to present a new family of weighted means for two values depending on a parameter $k$. These means possess specific properties which are essential in different applications. As examples of this kind of uses, we mention subdivision and multiresolution schemes over nonuniform meshes and the numerical solution of conservation laws. Specifically, we deal with three main properties of the means: to be bounded by a constant times the minimum of the two values, to give a close value to the weighted arithmetic mean of the two values under certain constraints, and to accomplish a Lipchitz condition. We prove that the defined family of means satisfy these requirements. We give a particular example of application to easily obtain accurate theoretical constants of stability in a slightly modified well-known nonlinear subdivision scheme.

References:


Splines cúbicos monótonos para datos procedentes de funciones discontinuas

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Abstract: La interpolación de Hermite, en particular la interpolación basada en splines cúbicos, ha sido utilizada en gran número de aplicaciones debido a las propiedades que presenta como suavidad y precisión del polinomio interpolante, p. ej. [1, 3]. Sin embargo, cuando existe una discontinuidad o los datos de los que partimos tienen un salto relevante, los interpoladores no se adaptan correctamente y aparecen fenómenos no deseados como el de Gibbs. En este trabajo presentamos dos algoritmos para evitar estos fenómenos produciendo interpolantes de Hermite monótonos [2]. Mostramos distintos experimentos numéricos que confirman los resultados teóricos.

References:


On error estimations for multiscale hybrid-mixed finite element methods on polyhedral meshes

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Abstract: Composite finite element H(div-)conforming vector (or tensor) approximations are considered for meshes allowing general polytopal subdomains, by keeping fixed the face normal traces constrained to a given finite dimensional trace space, piecewise defined over a partition of the mesh skeleton (facets). The approximations inside the subdomains may be enriched in different extents: with respect to internal mesh size, internal polynomial degree, or both. Their implementation requires a hierarchy of vector shape functions already available for classic polynomial spaces of arbitrary degree defined on usual master elements. The construction of such constrained space settings are useful for several applications, as for conformal combination of different element geometry in the same mesh, or for the design of hp-adaptive schemes. Recently, they have also been used in the design of stable multiscale hybrid-mixed formulations (denoted by MHM-Hdiv for Darcy’s flows [1], and by MHM-WS for linear elasticity problems weakly imposing stress symmetry [2]). A priori error analyses of these methods demonstrate accuracy order for the flux, stress and stress symmetry determined by the trace discretizations, despite the resolution increments inside the subregions. Moreover, enhanced accuracy rates for pressure (or displacement) and super-convergent divergence of the flux (and of the stress) can be obtained. Some verification numerical experiments shall be presented to illustrate the efficiency of the proposals.

References:


An improved model for the drying of solids with abruptly changing porosity

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Abstract:

We consider the Darcy-Forchheimer model for the simulation of the drying of solids [4]. This model arises in fluid mechanics through porous media at moderate or high velocities, and can be useful to predict the drying rate of the first drying period, which is usually constant [6]. It is common that porosity undergoes abrupt changes that create strong singularities; these make it difficult to obtain an accurate prediction of the fluid flow, velocity and pressure drop [5]. In order to overcome this difficulty, we make use of a simple a posteriori error indicator, reliable and locally efficient, that allows us to refine the mesh over the singularities [1]. We also elaborated a code in FreeFEM++ [2, 3] which enabled us to perform some numerical experiments that confirm the theoretical results.

References:


Integration of the source term in transcritical rarefactions when using approximate Riemann solvers.

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Abstract: The Roe solver, with its augmented version, is one of the most popular approaches to solving numerically the hyperbolic conservation laws in Fluid Mechanics. In Roe’s method, the solution of the Riemann problem (RP) is based on linearized eigenvalues of the homogeneous part of the system [1], which excludes the presence of the source term. The augmented Roe (ARoe) solver includes the effect of the source terms over the inner states of the approximate solution, which is vital in many problems, from one dimensional (1D) flow in channels with arbitrary section [2], to pipes or vessels with variable mechanical and geometrical properties. This provides a clear path to ensure well balanced solutions. However, the Roe solver assumes that the RP can always be represented using shock-type solutions, compromising the resolution of transcritical rarefactions. One way to overcome this problem is to use a Harten-Hyman fix based on wave-splitting. In practice, the choice of the new waves inevitably demands new approximations that open up many possible paths, from the simplest option, where the wave celerities are estimated directly from initial conditions of the RP, to more complex approximations that seek to incorporate the effect of source term by bringing in the ARoe approach [3]. In the latter case, the choice of the source term integration has a huge impact on the solution [2, 3]. The effect of the numerical formulation is analyzed in this work throughout several approximations. A solid and methodical formulation of the problem is proposed and approximations are unraveled. Numerical test cases are used to compare their accuracy.

References:


An a posteriori error analysis for incompressible elasticity problem with mixed boundary conditions

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- Rommel Bustinza (rbustinza@udec.cl) Universidad de Concepción, Chile

Abstract: In this talk, we propose an augmented finite element formulation for an incompressible elasticity problem. We consider mixed boundary conditions: null displacement on some part of the boundary (Dirichlet), as well as a given traction on the other part (Neumann). First, we assume the situation where the traction on Neumann boundary is also zero. By introducing the pseudo-stress as an additional unknown, we derive an augmented variational formulation for the pseudo-stress and the displacement, at continuous and discrete levels. We prove that the scheme is well-posed, stable and convergent. After that, we deduce an a posteriori error estimator, which is reliable and efficient. As a by product, we are able to extend and adapt the described procedure to the case where the traction on Neumann boundary is given and is non null. Finally, we show some numerical tests, that are in agreement with our theoretical results.

References:


New families of eighth-order Runge–Kutta–Nyström symplectic integrators

Authors:
• Alejandro Escorihuela-Tomàs (alescori@uji.es) Universitat Jaume I

Abstract: We present different families of new Runge–Kutta–Nyström (RKN) symplectic splitting methods of order 8, especially designed for second-order systems of ordinary differential equations of the form $y'' = f(y)$. They show a better efficiency than state-of-the-art symmetric compositions of 2nd-order symmetric schemes and RKN splitting methods of orders 4 and 6 for medium to high accuracy. For some particular examples, they are even more efficient than extrapolation methods for high accuracies and integrations over relatively short time intervals, whilst preserving qualitative features of the system.

We analyze in detail the construction process: it is based on the use of the Lie formalism to obtain the order conditions, what are then numerically solved by using continuation methods implemented in Python.

This is a joint work with Sergio Blanes (Universitat Politècnica de València) and Fernando Casas (Universitat Jaume I).

References:
Numerical analysis of a poro-elastic MGT problem

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- Ramón Quintanilla (ramon.quintanilla@upc.edu) Universitat Politècnica de Catalunya

Abstract: In this communication, we analyze, from the numerical point of view, dynamic problem arising in thermoelasticity by using the so-called Moore-Gibson-Thompson theory (see [2]). Dielectrics effects are also included within the model. The corresponding problem is written in terms of the displacement field, the temperature and the electric potential. We recall an existence and uniqueness result recently proved in [1]. Then, a viscous term is added in the heat equation to provide the numerical analysis of the corresponding variational problem. Then, by using the finite element method and the implicit Euler scheme fully discrete approximations are introduced. A discrete stability property and a priori error estimates are obtained. Finally, one-dimensional numerical simulations are shown to demonstrate the accuracy of the approximation and the behavior of the solution.

References:


Evaluation of a general car-following model for micro/macro traffic modelling

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Abstract:
In transport traffic analysis, road traffic is modelled as a system of interacting particles. In the last decades a number of mathematical models have been proposed to represent the dynamics of this process. Most important among them are the car-following models, which have been used to model both the inter-vehicles dynamics (micro modelling) and the aggregated traffic process (macro modelling).

Even though these models accurately represent phenomena observed in real roads, like congestion creation, metastability and wave propagation, they are sometimes based on narrow assumptions that do not match the behaviour of real drivers and vehicles. The accuracy of these models is specially relevant now that new communication technologies like 5G and V2X (Vehicle to everything) are intended to be a key part of the future road-traffic infrastructure, and will require stringent latency requirements, complicating the dynamics of the traffic process.

This paper will present results on a new, general purpose traffic model to be used in the study of the impact of new V2X technologies on the road traffic phenomena. The results to be presented are along the following lines:

- Challenges for an unified model for micro/macro traffic simulations.
- Applicability to micro (for particle dynamics) and macro (for fluid-like dynamics) simulation scenarios.
- Numerical evaluation and validation of the mathematical model using an extension to network simulator ns-3.
- Future developments related to self-driving traffic management and its interaction with the supporting communications technologies.

References:
Boundary integral methods for simulating scattering and transmission problems in 2D elastodynamics

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Abstract: We present some new robust formulations in Boundary Integral Methods for elastodynamics (time-harmonic) scattering and transmission problems on smooth bidimensional domains with piecewise constant material properties. We show how simple, easily-evaluable approximations of Dirichlet-To-Neumann operators can be used along with Calderon calculus to yield well-posed integral equations of the second kind for the corresponding boundary partial differential problems. That is, the underlying operators in the continuous equations are invertible compact perturbations of the identity. As a direct consequence, once discretized, the linear systems are expected to have matrices with bounded condition number, regardless of the level of discretization, and with eigenvalues clustered around 1, which makes them more amenable to be solved by iterative solvers such as Krylov’s methods. Furthermore, although both the condition number and the eigenvalues distribution still depend on the frequency of the problem, this dependence is shown to be milder.

The numerical solution of the boundary integral equations is carried out with a superalgebraic convergent Nyström-type scheme, which follows the ideas first introduced by Kress for Helmholtz equation [6, Chapter 12] (see also [2]). The algorithm hinges on a precise splitting of the kernels of the integral operators into smooth and singular parts, an accurate description of these singularities and an application of product quadrature rules for each type of the integrals that arise after this splitting. This approach, although relatively straightforward from a theoretical point of view, relies heavily on a very detailed description of the $2 \times 2$ fundamental solution matrix function of the elastodynamics, as well as its first derivatives, which is also discussed here.

Finally, we refer the reader to [4] for a thorough exposition of the results of this talk.

References:


Solving linear and nonlinear PDEs in the exascale using Monte Carlo

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Abstract: Probabilistic Domain Decomposition (PDD) is a non-standard, highly-scalable algorithm for large-scale boundary-value problems (BVPs), in which the subdomains are decoupled by first calculating the solution on the fictitious interfaces thanks to probabilistic representations of BVPs [1]. Nevertheless, the complexity of the probabilistic representations of most non-linear BVPs [2] uses to hinder the usage of PDD to solve them.

In order to overcome this difficulty, we have developed nonlinear PDDSparse: A new iterative algorithm aimed at computing the solution of large-scale elliptic BVPs in modern supercomputing environments. In our approach, a nonlinear BVP is recast into a sequence of linearised ones via Fréchet Calculus [3]. Then, each linear problem is solved using PDDSparse, our brand new version of PDD.

In this talk, we will present PDDSparse and nonlinear PDDSparse. Alongside their formulation, we will discuss some details of their implementation and show some of the tests performed at Marconi-100, a GPU accelerated supercomputer located in the CINECA supercomputer facility in Italy.

References:
Meshless finite difference method for solving fractional differential equations at irregular meshes

Authors:
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Abstract: In this communication we consider the fractional differential equation in the interval $[0, 1]$:

$$
\begin{align*}
\beta \frac{\partial u}{\partial t} &= L(u) + \mathcal{L}(u) + h(u, t, x), \quad x \in [0, 1], \quad t > 0, \\
u(t, 0) &= u_L, \quad u(t, 1) = u_R, \quad t > 0, \\
u(0, x) &= u_0(x), \quad x \in [0, 1],
\end{align*}
$$

(1)

where $L$ is

$$
L := \frac{f_1(u, t, x)}{\partial x} + \frac{f_2(u, t, x)}{\partial x^2}
$$

(2)

and

$$
\mathcal{L} := g(u, t, x) D_{\alpha}^a.
$$

(3)

This paper presents a novel meshless technique for solving a class of fractional differential equations based on moving least squares and on the existence of a fractional Taylor series for Caputo derivatives, given in [2] among others. A "Generalized Finite Difference" approach is followed in order to derive a simple discretization of the space fractional derivatives. Consistency, stability and convergence of the method are proved. Several examples illustrating the accuracy of the method are given. This results are partly covered at [1].

References:


A deep neural network for solving PDEs using r-adapted meshes

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Abstract: The use of Deep Neural Networks (DNNs) to solve Partial Differential Equations (PDEs) has risen great interest during the last decade. Within this context, some traditional methods have been adapted to DNNs, resulting in the Deep Ritz method [1], the Deep Least-Squares method, and the Deep Galerkin method, among others. Some of these methods are capable of solving PDEs in high dimensions using a Monte Carlo type integration. However, mesh-based quadrature rule are much more efficient in low dimensions. Unfortunately, the use of fixed quadrature points may produce overfitting on training. In [2] we address this challenge and propose several techniques to overcome it.

Herein, we propose a deep r-adaptive method to simultaneously optimize both the piecewise-polynomial approximation of the PDE’s solution and the location of the nodes in a mesh with a tensor-product topology. Our method does not require a posteriori error estimation and adapts the mesh during training.

To numerically illustrate the performance of our proposed r-adaptive method, we apply it in combination with the Deep Ritz Method. We show the behavior in one- and two-dimensional domains for solutions that are smooth, singular, or have a strong gradient.

References:


Adapting cubic Hermite splines to the presence of singularities through correction terms

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Abstract:
Hermite interpolation is used to reconstruct smooth data when the function and its derivatives are available at certain nodes. If derivatives are not available, it is easy to set a system of equations imposing some regularity conditions at the data nodes in order to obtain them. This process leads to the construction of a Hermite spline. The problem of the described Hermite splines is that the accuracy is lost if the data contains singularities. The consequence is the apparition of oscillations, if there is a jump discontinuity in the function, that globally affects the accuracy of the spline, or the smearing of singularities, if the discontinuities are in the derivatives of the function. This work is devoted to the construction and analysis of a new technique that allows to obtain accurate derivatives of a function close to singularities using a Hermite spline. The idea is to correct the system of equations of the spline in order to attain the desired accuracy even close to the singularities. Once we have computed the derivatives with enough accuracy, the addition of a correction term to the cubic Hermite spline in the intervals that contain a singularity, allows to reconstruct piecewise smooth functions with $O(h^4)$ accuracy even close to the singularities. The process of adaption will require some knowledge about the position of the singularity and the values of the function and its derivatives at the singularity. The whole process can be used as a post processing, where a correction term is added to the classical cubic Hermite spline. The numerical experiments presented confirm the theoretical results obtained.
A finite difference approximation scheme for the self-adjoint extensions of singular Sturm-Liouville operators

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Abstract: In physics, the self-adjoint extension of a symmetric linear operator is uniquely determined by the physical conditions of the problem. In the case of Sturm-Liouville operators the self-adjoint extensions are given by the boundary conditions at one or two points. Due to the difficulties in manipulating the boundary conditions at isolated points, in Quantum Mechanics only the Friedrichs extension is considered [1]. In other systems (magnetic materials, for instance), the recent technological advances could allow us to select different boundary conditions and other extensions may become relevant.

In this communication I present a finite difference approximation scheme to study the eigenvalue problem $Lu = \lambda u$, with $u(r)$ defined in $[0, R]$ and $L$ acting as

$$Lu = -u'' + \frac{\nu(\nu - 1)}{r^2} + \frac{q}{r} + v(r)u,$$

(1)

where $1/2 < \nu < 1$, $q$ is a real number, and $v(r)$ a function analytic at $r = 0$. Functions $u$ of the domain of $L$ satisfy the boundary conditions $\lim_{r \to 0}(uu'_\alpha - u'u_\alpha) = 0$ and $u(R) = 0$, where $u_\alpha(r) = \cos \alpha r^\nu + \sin \alpha r^{1-\nu}$. The singularity of $L$ at $r = 0$ makes the standard three point central difference approximation for the second derivative not consistent.

A consistent and convergent scheme, of second order, for the Friedrichs extension ($\alpha = 0$) was proposed long ago by Dershem (for the Euler equation), and by Reddien (for the operator $L$), and more recently by Laliena and Campo for the spin wave operator in presence of Skyrmions [2]. For $\alpha \neq 0$ the finite difference approximation is much more complicated and has not been studied yet.

In this communication I show that a consistent scheme, of order lower than two, is obtained by imposing appropriate conditions on a generic three point finite difference approximation of the second derivative. I also proof the convergence of the eigenvalues of the discretized problem to those of (1). Some numerical experiments which illustrate these ideas are also presented.

References:
Numerical computation of the fractional Laplacian with $\alpha = 1$ by means of Gauss hypergeometric functions.

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Abstract: We propose an optimal numerical method to calculate the following Gauss hypergeometric functions:

$$\ _2F_1(1, n/2 - p_1; n/2 + p_2; z),$$

for an odd number $n$, and for $|z| \leq 1$, $z \neq \pm 1$, $p_1, p_2 \in \mathbb{N} \cup \{0\}$, such that $p_1 + p_2 > 0$.

The purpose of this implementation is the numerical computation of the one dimensional fractional Laplacian applied to Fourier modes. More precisely, the change of variable $x = L \cot(s)$, with $L > 0$ and $s \in [0, \pi]$, maps $\mathbb{R}$ into a finite interval where spectral or pseudo-spectral methods might be performed. Indeed, we have obtained the following representation for the fractional Laplacian in this new variable with $\alpha = 1$ applied to elementary trigonometric functions:

$$(-\Delta)^{1/2} e^{ins} = \frac{-2i}{L \pi (n-2)} + \frac{8i}{L \pi (n^2 - 4)} \ _2F_1(1, n/2 - 1; n/2 + 2; e^{-2is}) + \frac{ne^{ins} \sin^2(s)}{L},$$

for an odd number $n$, so that with this formula we are able to approximate the fractional Laplacian of $u(x(s))$ when $\alpha = 1$ with the Fourier series expansion.

As an application, we carry out a simulation of the defocusing fractional cubic nonlinear Schrödinger equation with $\alpha = 1$:

$$\begin{cases}
i \frac{\partial \psi}{\partial t} = \frac{1}{2} (-\Delta)^{1/2}\psi - |\psi|^2\psi \\
\psi(0, x) = \psi_0(x)
\end{cases}$$

References:

P4. Numerical Linear Algebra
Totally positive matrices, Bernstein polynomials, and applications to CAGD

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Abstract: The relationship between total positivity and shape preserving properties of bases (convex hull property, endpoint interpolation, variation diminishing property) is a classical subject that has been analyzed in [1], although these results are not usually presented in general books of CAGD, such as the well-known text [2], surely due to a lack of appreciation of linear algebra.

The optimality results about the Bernstein basis presented in [1] inspired the work [4], where total positivity was exploited to construct fast and accurate algorithms, based on the bidiagonal decomposition of Bernstein-Vandermonde matrices (a name given in [4] to the collocation matrices of the Bernstein basis), for computing with those matrices. In that paper the explicit form \( y = f(x) \) of a function is considered, and the problem being solved corresponds to polynomial interpolation.

Now we return to the parametric setting \((x(t), y(t))\) corresponding to Bézier curves and we consider, in addition to interpolation, the case of least squares fitting with Bézier curves. The algorithms developed for this problem are also applied to the degree reduction problem (in the context of Bézier curves), an issue addressed in Section 5 of [3] and in Chapter 6 (titled Bézier Curve Topics) of [2].

References:

On the best approximation of square matrices in Laplacian-like form

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Abstract:
Linear systems are widely used to approach computational models in applied sciences, for example, in mechanics after the discretization of a partial differential equation. There are numerous numerical approaches to compute the solutions of these types of problems; however, most of them lose efficiency as the size of the matrices or vectors involved increases. This effect is known as the curse of dimensionality problem.

To try to solve this drawback, we employ tensor-based algorithms since their use significantly reduces the number of operations that we must employ. From the study of the GROU procedure to solve high-dimensional linear systems, we observe that there is a type of matrices for which the algorithm works particularly well: very fast convergence and a very good approximation of the solution. These are the so-called Laplacian-Like matrices, which have the form

\[ A = \sum_{i=1}^{d} \text{id}_{[n_i]} \otimes A_i = \sum_{i=1}^{d} \text{id}_{n_i} \otimes \ldots \otimes \text{id}_{n_i-1} \otimes A_i \otimes \text{id}_{n_i+1} \otimes \ldots \otimes \text{id}_{n_d}. \]

Starting from this fact, we propose a ‘Laplacian decomposition’ for square matrices to improve the convergence of the linear system associated, and describe the procedure in algorithm form.

References:
P5. Optimal Control - Inverse Problems
Optimal control for cardiovascular diseases

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Abstract:
Fluid control presents great challenges. Specifically, in this case, we seek to control a fluid representing blood flow. This is particularly relevant to the understanding of some cardiovascular diseases such as an aneurysm, an arterial obstruction, etc [1]. We present some results for the case of the Navier-Stokes equations. In fact, we solve a boundary optimal control problem for the evolutionary Navier-Stokes equations with mixed Dirichlet-Neumann boundary conditions, both in two and three dimensions. Following previous work,[2, 3], we provide additional details about the theoretical and numerical study of the solution of the boundary control problem associated with the Navier-Stokes equations under more realistic assumptions.

References:
Imaging electromagnetic scatterers in the time domain

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Abstract: We study the reconstruction of the shape of a bounded scatterer from time domain electromagnetic scattering data measured on a surface containing the unknown scatterer and due to (regularized) magnetic dipoles located away from the scatterer. This inverse problem is non-linear and ill-posed. To decrease the need for a priori data and mitigate the computational burden, instead of an optimization based scheme we opt for a qualitative method that seeks to determine the shape of the scatterer without determining its material properties. Indeed, the time domain linear sampling method (TD-LSM) builds an approximate indicator function of the scatterer at some sampling points (in a region where the scatterer is located) by solving a sequence of linear integral equations (the so-called near-field equation). To date, the method has been used for the acoustic wave equation and has been tested for several different types of scatterers (sound hard, impedance, and penetrable), and for waveguides. Our work is the first application of the TD-LSM in electromagnetism: Based on the Laplace transform approach previously used for the acoustic wave equation, we analyze the use of the TD-LSM to the time dependent Maxwell’s system with impedance boundary conditions, and a similar analysis applies for a perfect electric conductor (PEC). Our results support the use of the TD-LSM for this problem, and we provide preliminary numerical tests of the algorithm that confirm our analysis.

References:

Null controllability for linear parabolic equations. Applications to therapies.

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Abstract: The null exact controllability with a partially distributed control consist of finding a function $u^*$, called a control, which acts on a subset $\omega$ of the domain $\Omega$, such that the solution of the partial differential problem, in a fixed time $T$, is zero. The paper [1] studies this issue for the semilinear heat equation, using Carleman’s inequalities. The proofs of this kind of inequalities require regular coefficients in the second order partial derivatives. There are important models which are stated by parabolic equations with non-regular coefficients like the spread of a brain tumor where diffusion coefficients are non-continuous (see [3]). The question we answer in this work is how to get a control which, acting in a little part of the domain, leads the solution of a general linear parabolic problem, to zero in a time $T$. The main result is the following: Let be $u \in L^2(\omega \times (0, T))$, $u \geq 0$, $y_0 \in L^2(\Omega)$, $y_0 \geq 0$, $A \in L^\infty(\Omega)^{N \times N}$ a positive definite matrix. If we denote by $\Psi_u$ the solution of the parabolic problem $\partial_t - \nabla \cdot (A(x) \nabla)$, with right member $u$, homogeneous Dirichlet boundary conditions and initial data $y_0$, $\| \|$ the norm in $L^2(\Omega)$ then, there exists a function $v^* \in L^2(\omega \times (0, T))$, $v^* \geq 0$, such that

$$u^* = \frac{\| \Psi_{v^*}(T) \| u - \| \Psi_u(T) \| v^*}{\| \Psi_{v^*}(T) \| - \| \Psi_u(T) \|},$$

is a null exact control, i.e. $\Psi_{u^*}(T) = 0$.

In applications, we obtain the control for two models to simulate the dynamic of a brain tumor (see [2]). The simulations say to us that it is necessary more therapy at the beginning than at the final time, the control function is continuous in time and the spatial domain where it acts is the support of tumor cell density.

References:


Comparison of the topological derivative behavior on different scenarios: the time-harmonic heat and Maxwell equations.

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Abstract:

The topological derivative [3] has been successfully used for inversion problems on a wide variety of physical phenomena. On the one hand, in [2] two of us studied its application to a thermal problem with numerically generated data. The problem consisted in finding cavities in an aluminum plate given several thermograms of one of its surfaces. These thermograms were obtained while the plate was heated with an infrared lamp in a time-harmonic manner. On the other hand, in [1] we studied its application to the time-harmonic Maxwell equations as well as its simplification to the Helmholtz equation. Experimental data from the Fresnel database was used, which consists of a set of experiments where several conducting or dielectric objects were irradiated with time-harmonic electromagnetic waves while the electric field was measured on a set of antennas.

In this talk we will discuss both situations, showing some key differences in the results that can be expected from each of the phenomena, as well as comparing the usage of numerical and experimental data. Both the inversion algorithm as well as the formulas used to compute the topological derivative shared some parallelism between the two phenomena, which was very helpful as some parts of the code and ideas could be reused. Each one of the experiments presented its own advantages and limitations. We will finish the talk by commenting on some of the possible solutions to these problems, which are tailored to each of the phenomena.

References:

Numerical methods for object detection in attenuating media

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Abstract:
In this work we aim to detect objects hidden in a medium from measurements of the acoustic total wave field recorded at a few detectors. This is an inverse problem in opposition to the forward problem that describes the behaviour of the acoustic waves in the presence of known objects.

Most inverse algorithms for object detection from acoustic data assume that the attenuation coefficient is negligible. However, in some applications in subbottom tomography or in medical studies of biological tissues, attenuation effects cannot be discarded.

Numerical methods based on the computation of the topological derivative to reconstruct objects buried in attenuating media from limited aperture data will be proposed. Emitters and receivers will be located on the accessible part of the sounded region, which complicates detection to a great extent. We will show that one-step implementations of the algorithm provide initial approximations, which are improved in a few iterations. A discussion about the capabilities and limitations of the methods will be presented.

References:

Parabolic ejection & collision orbits for the restricted planar circular three body problem

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Abstract:

We consider the restricted planar circular three body problem (RPCTBP), which describes the motion of a massless body under the attraction of other two bodies, the primaries, which describe circular orbits around their common center of mass located at the origin. In a suitable system of coordinates, this system has two degrees of freedom and the conserved energy is usually called the Jacobi constant. We are interested in solutions of the RPCTBP that collide with the big primary at some instant $t_0$. We will distinguish between an ejection orbit (that is, the particle kicks out from collision with the big primary at some instant $t_0$) and a collision orbit (the particle goes to collision with the big primary at some instant $t_1$). In particular, we will study the case when both ejection and collision orbits travel close to infinity for small values of the mass ratio.

To obtain such orbits, we show that, for small values of the mass ratio and the Jacobi constant, there exist transversal intersections between the stable (unstable) manifold of infinity and the unstable (stable) manifold of the collision.
Bernstein-type Operators based on the Jacobi inner product

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Abstract:
Bernstein polynomials were introduced by S. Bernstein in 1912 to provide a constructive proof of the Weierstrass approximation theorem. In this way it was established that every continuous function defined in the interval $[0, 1]$ can be uniformly approximated by Bernstein polynomials in such interval.

In this work we study a modification of the Bernstein operator that was studied in [2] by means of the Jacobi inner product. We analyze its properties on different types of functions and their possible applications.

References:


P6. Applied Mathematics to Industry, Social Sciences and Biology
A mathematical model for the energy stored in green roofs

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Abstract: Poor air quality, increased building energy consumption and the heat island effect are examples of environmental problems associated to urbanization. Green roofs (i.e., roofs partially or totally covered with vegetation) are a particularly attractive solution to mitigate some of these problems. Their capacity to reduced heating/cooling energy demand, increase the rainwater retention and reduce sewerage loads, or enhance biodiversity in cities, are well documented [1, 2, 3]. Modelling green roofs is a relatively difficult task due to the interplay between sun radiation, air convection, water evaporation and the role of plants in the energy and water mass balance of the whole system. In this talk, we will present a simple mathematical model for predicting the temperature evolution and the energy stored in a green roof during daytime. The model contains all the essential physical processes acting on the roof, and it is simple enough to allow the use of Laplace transforms and the eigenfunction expansion technique to produce exact solutions and practical formulas to assess the green roof performance.

References:

A degenerating convection-diffusion system modelling froth flotation with drainage

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Abstract: Froth flotation is a common unit operation used in mineral processing. It serves to separate valuable mineral particles from worthless gangue particles in finely ground ores. The valuable mineral particles are hydrophobic and attach to bubbles of air injected into the pulp. This creates bubble-particle aggregates that rise to the top of the flotation column where they accumulate to a froth or foam layer that is removed through a launder for further processing. The drainage of liquid due to capillarity is essential for the formation of a stable froth layer at the top of the column. This effect is included into a previously formulated hyperbolic system of partial differential equations that models the volume fractions of floating aggregates and settling hydrophilic solids [1].

We will detail the construction of desired steady-state solutions with a froth layer and provide algebraic equations and inequalities that establish the dependence of steady states on the input and control variables. Such dependences are conveniently visualized in so-called operating charts that constitute a graphical tool for controlling the process. The particular importance of steady states comes from the application under study; namely they describe the ability of the model to capture steady operation of the flotation device without the necessity of permanent control actions. A monotone numerical scheme for the new governing PDE system will be presented and employed to simulate the dynamic behaviour of a flotation column. It is also proven that, under a suitable Courant-Friedrichs-Lewy (CFL) condition, the approximate volume fractions are bounded between zero and one when the initial data are.

References:

Time parallel methods for epidemiological models

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Abstract: Epidemiological models have been a source of great interest in last years. In general, they are complex models in which the time evolution of many kinds of individuals is taken into account. However, variation in the spatial distribution of these individuals is not usually considered. Even less, PDE models are used for continuous space variation of a population. A reason for this disassociation would be the complexity of these models along with the huge computational effort that they requiere.

In this work, we propose a spatial SIR model in which diffusive and advective evolution are contemplated. Nonetheless, it is well known that the development of numerical schemes for this kind of equations is a really difficult task. Our purpose is to introduce Galerkin schemes in space with flux correction [1], a technique born in last decades to keep the maximum principle in convective problems. On the other hand, to reduce the computational time required, we use Parareal, a time parallel method, which was introduced by Lions et al. in 2001 [2].

References:

Computing $\Delta V$ for orbital maneuvers between non-coplanar and non-coaxial elliptical Keplerian orbits

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Abstract:
In the aerospace industry, it is needed to apply impulsive maneuvers in different situations like rendezvous, docking, repositioning of any satellite, to put a spacecraft into exploration launch traffic and others [1, 2].

Nowadays there is a wide spectrum of possibilities to modify the trajectories of the orbiters and thus generate new orbits. Modification of the trajectory in space of any orbiter usually requires the highest possible precision to allow the success of any mission. Different orbits can be designed, however today it is more complicated because there are many orbiters, such as space debris that endangers any space mission and therefore it is necessary to change the orbit. The application of impulsive maneuvers is fundamental but they are restricted by a set of requirements.

This research aims to present a fundamental energy cost function, $\Delta V$, taking into account that the initial and final orbits are non-coplanar and non-coaxial elliptical Keplerian orbits. This particular function is obtained as a function of two orbital elements of the transfer orbit, semimajor axis and eccentricity, regardless of the two impulsive maneuvers that are applied. Three elementary configurations derived as a direct consequence of the general formulation are studied. In this way, the considered impulsive maneuvers that involve coplanar and non-coplanar orbits, as well as coaxial and non-coaxial orbits, make possible a change from an inner orbit to an outer orbit. Finally, various sets of orbital elements are used to assess the cost of energy and illustrate the different configurations.

References:
Bow appendages in small fishing vessels. Resistance analysis

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Abstract:
Nowadays, it is usual the installation of different types of bow appendages in small fishing vessels, always under the request of the shipowners who affirm to reduce fuel consumption. Mainly, this fuel saving is observed in artisanal small fishing vessels, which have as common characteristic a waterline length between 6 and 10 meters. This type of vessel does not fall within the typical ranges of waterline length in which it is advisable to install bow bulbs. Also, these appendages can have different shapes and do not follow standard types.

In this work, we analyze this good behavior due to the decrease of the total resistance to the advance of small fishing vessels by the incorporation of the bow appendages. For this, the values for the total resistance between hulls with appendages and without appendages will be examined and compared using Galerkin methods. Specifically, we will examine the use of splitting finite-elements schemes for the numerical simulation of the flow around the hull of the ship. This is not an easy task because in this case we have to solve partial differential problems, type Navier-Stokes, with difficult-to-treat Reynolds and Froude numbers in complex 2d and 3d geometries.

References:
Microwave irradiation and conventional heating: a comparison using in silico experiments with water and ethylene glycol

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Abstract: In this work we compare numerically the behavior of two solvents, water and ethylene glycol, under two different heating mechanisms: microwave irradiation and conventional heating. The thermophysical and dielectric properties of each solvent determine how they are heated up. Under conventional heating, the temperature profiles show the development of thermal plumes at the bottom that move towards the lateral wall, being convection the mechanism heating up the sample. For ethylene glycol, of higher viscosity than water, the number of thermal plumes is considerably reduced implying a slower heating of this solvent in comparison to water. However, under microwave irradiation, ethylene glycol heats faster than water due to its higher susceptibility to microwave irradiation. Temperature profiles show stratification and the localization of a hotter region at the centre of the sample for both solvents. A three dimensional temporal model is used coupling heat, momentum equations and Maxwell equations based on spectral element methods. Results are interesting as they provide, in silico, a full spatio-temporal description of temperature and flow velocity of the solvents under these two heating mechanisms.
Model transform and local parameters. Application to instantaneous attractors

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Abstract: The model transform (MT) fits exactly the parameters of a suitable model to empirical or simulated data in each point in time and/or space. The MT allows simple theoretical models to be applied to complex empirical systems in each short interval of time or/and in each local neighbourhood. The model can be chosen to identify, for instance, the temporal evolution of the attractor landscape for empirical systems which depict a complex dynamics over time.

1. We propose an alternative to the classical approach to modeling complex systems. The MT is a generalization of the transformation performed by trivial models to find new measurements when applied to non-trivial models and characterize different states of the most complex and difficult systems to model.

2. For instance, the novel mathematical formalism we propose can characterise how the attractors of a dynamical system evolve in time. Instead of describing dynamics in terms of asymptotic behaviour, we introduce the instantaneous global attractors and their dynamical structures, the directed graphs associated with the attractors.

3. It has been applied to distinguish quantitatively between different human brain states ([1, 2, 3]), using a dynamical model that integrates the underlying anatomical structure with the local dynamics.

4. In Industry, Social Sciences and Biology, any model leads to new applications of the MT.

References:

Wind modeling applications: a case of study applied to the estimation of electricity production in wind farms

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Abstract: The fight against climate change and the need to reduce CO₂ emissions have led to the emergence of new sources of electricity generation in recent years. Among these new sources wind energy stands out, which differs from other traditional generation sources due to its variability and intermittency, inherent to the wind’s complex nature. For the correct working of the energy system, producers must provide a prediction of the energy they are going to produce during the following hours. This is a challenging task for wind producers, who have to face the unpredictable and intermittent behaviour of the wind.

In this work, we present a methodology to provide accurate short-term estimations of the production of electricity from wind sources, which consists of a downscaling procedure for obtaining more accurate weather forecasts through the coupling of a mesoscale model, also known as Numerical Weather Prediction (NWP), with the microscale wind model developed by authors HDWind [1]. In this sense, we use the NWP model to obtain meteorological predictions with adequate temporal scopes and we adapt them to the local characteristics of the terrain, such as topography or roughness, through the use of the microscale model HDWind [2]. HDWind is a local model that provides both a high-resolution wind field that covers the entire study domain and values of wind speed and direction at very located points [3], like the position of the different air turbines in a wind farm.

References:

An Inextensible Model for the Robotic Manipulation of Textiles

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Abstract: We introduce a new cloth model which models the dynamics of textiles as inextensible surfaces. This assumption challenges most models in literature where elasticity is allowed, sometimes by necessity (Textile Engineering) or in the pursuit of spectacularity (Computer Graphics). In [1], the inextensibility assumption is shown to be realistic by comparing simulations to experimental data, finding that the difference between the simulated and the real position of each point in garments is lower than 1cm on average.

Inextensibility is modeled as follows: we assume that our cloth $S$ is a surface (with boundary) moving through space whose metric (first fundamental form) is preserved. In order to implement these conditions (which are in fact PDE’s) on a computer, we assume that $S$ has been triangulated (or quadrangulated) and then apply a novel and non-trivial Finite Element discretization to the inextensibility constraints. If we denote by $\varphi(t)$ the position of the nodes of the polyhedron, this gives raise to a smooth (actually quadratic) constraint function $C(\varphi) = 0$ which must be preserved at all times. Making use of Signorini’s contact model, the dynamic equations of motion then would be:

$$
\begin{align*}
M\ddot{\varphi} &= F(\varphi, \dot{\varphi}) - \nabla C(\varphi)\lambda + \nabla H(\varphi)\gamma, \\
C(\varphi) &= 0, \\
H(\varphi) &\geq 0, \quad \gamma \geq 0, \quad \gamma^T \cdot H(\varphi) = 0,
\end{align*}
$$

where we have grouped in the force term $F$ damping, gravity, stiffness, aerodynamics, friction, etc. On the other hand, $H(\varphi) \geq 0$ contains the implicit equation of a given obstacle (e.g. a table) and in addition self-collision constraints. Since the collision and inextensibility forces turn out to be very stiff, the system must be integrated implicitly. This is done by solving a sequence of quadratic problems with linear constraints (basically by linearizing all non-linearities in (1)). In order to solve these quadratic programs efficiently, we develop a novel active-set numerical algorithm which takes into account which constraints were active from one iteration to the next. To our knowledge, our method is the first that results in a non-decoupled resolution of contacts, friction and inextensibility for cloth simulation in a single pass.

References:

Improving mathematical simulations through high quality input data: a case of study applied to the simulation of wildfires

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Abstract: An environmental model is a description of a phenomenon of the environment using mathematical concepts and language. These models may be used purely for research purposes, or for providing a real-world analysis to support decision making and policy. In this sense, wildfire modelling [1] allows the simulation of forest fires taking into account the local characteristics of the terrain where they may take place, such as the slope, the fuel load and type, weather conditions, ignition location or fire suppression tactics; and predicts the fire spread behavior. The accuracy of the prediction depends on the simplifications that the mathematical model assumes, as well as, the goodness of the input data that feed the model. These data are mostly managed by Geographic Information Systems (GIS) [2], which usually provides the data required by the simulation models. Unfortunately, these data sources often present problems related to a lack of updating or completeness for all areas, which impair the simulation results. In wildfire simulations, this is a common problem that affects the data related to the characterization of forest fuel. In this work we propose the use of artificial intelligence algorithms together with remote sensing technologies to deal with those problems [3]. The proposed procedure has been tested on an area located in Northeast of Castilla y León. We have classified the surface vegetation for different fuel types, according to Rothermel criteria adapted to the vegetation on the Iberian Peninsula. We have evaluated the procedure's performance through several validation metrics applied over different test sets in a cross-validation process. The results record an accuracy of close to 78%, improving some of the results reported in previous studies [4]. The final goal behind the improved data will be to enhance the simulations results where they are used.

References:


Immersed boundary approach to biofilm spread on surfaces

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- Ana Carpio (carpio@mat.ucm.es) Department of Applied Mathematics, Universidad Complutense de Madrid, Spain, and G. Millán Institute, Universidad Carlos III de Madrid, Spain

Abstract: We propose a computational model to study the growth and spread of bacterial biofilms on interfaces, as well as resistance to antibiotics [1]. Bacterial membranes are represented by boundaries immersed in a fluid matrix and subject to interaction forces [2]. Growth, division and death of bacterial cells follow dynamic energy budget rules, in response to variations in environmental concentrations of nutrients, toxicants and substances released by the cells, [3]. In this way, we create, destroy and enlarge boundaries, either spherical or rod-like. Appropriate forces represent details of the interaction between cells, and the interaction with the environment. Numerical simulations illustrate the evolution of top views and diametral slices of small biofilm seeds, as well as the action of antibiotics. We show that cocktails of antibiotics targeting active and dormant cells can entirely eradicate a biofilm. The extensive published work can be found in reference [4].

References:
Models and numerical methods for pricing renewable energy certificate derivatives

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Abstract:
In this work we state the mathematical model that governs the valuation of derivatives whose underlying is a REC, in particular we study European options and futures contracts. Assuming that the price of the renewable energy certificate (REC) or green certificates ([1], [3]) is known, we formulate the valuation problem of a European REC derivative in terms of a linear PDE model where the underlying stochastic factors are the accumulated green certificates sold by an authorized producer and the natural logarithm of the renewable generation rate. Next, a result of existence of solution [4] is obtained for this PDE problem. In order to solve numerically the PDE problem, we propose a Lagrange-Galerkin scheme for time and space discretizations ([2]). Finally, we show some numerical results that illustrate the performance of the proposed model and the numerical methods.

References:
Coupled models for equilibrium problems with heterogeneous agents

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Abstract: In this talk, we assume rational expectations to pose general equilibrium models with heterogeneous firms, which can enter or exit the industry [2]. More precisely, we consider a general Itô process for agent productivity dynamics. A Hamilton-Jacobi-Bellman (HJB) formulation models the endogenous decision of firms to remain or leave the industry [1]. All firms that exit are immediately replaced by a group of new ones, so that the probability density function of firms satisfies an appropriate Kolmogorov-Fokker-Planck (KFP) equation with source term [4]. Equilibrium models are completed with the household problem and feasibility conditions. In order to solve the equilibrium problem, we propose a Crank-Nicolson scheme in time. Moreover, we use an augmented Lagrangian active set (ALAS) method [3], jointly with a finite difference discretization for the HJB formulation and an adequate finite difference method for the KFP equation. For the global problem, we develop a Steffensen algorithm. Finally, numerical examples illustrate the performance of proposed numerical methodologies as well as the expected behaviour of computed economic variables [5].

References:
Bayesian mechanistic model of COVID-19 transmission dynamics including the effect of vaccination

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Abstract:

In our group, we have been working on optimizing a previously reported COVID-19 transmission model, which has been extended here for the analysis of multiple periods of different transmission rates, enabling the inclusion of an arbitrary number of non-pharmacological measures. Additionally, the model has been extended to include the effect of vaccination and the impact of the different virus variants on the transmission dynamics.

The algorithm computes the evolution of the daily number of infections by fitting a SEIR model to the observed daily deaths, in a Bayesian framework, using MCMC optimization to obtain the a posteriori distribution for the parameters that best describe the impact on the transmission rate of each intervention measure. The model captures the positive impact of the vaccination on the evolution of the disease, takes into account the immunity of the recovered population and considers specific transmission parameters for the different virus variants.

The model was successfully applied to a total of 30 European countries, obtaining good fit results and conclusions related to the impact of the different interventions that were consistent with results from other studies.

Interestingly, the model also estimates the percentage of immune population required to reach the herd immunity in the different countries, which is a valuable tool to understand the evolution of the pandemics on the long term and help in future worldwide control strategies.

References:


Towards large-scale sustainable water photo-electrolysis: modelling the electrochemical behaviour of titanium dioxide electrodes

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Abstract: Solar photoelectrolysis of water is a promising means for green hydrogen production. Cost effectiveness and efficiency remain major challenges to its wide industrial application. Such a goal requires in-depth understanding of the underlying photoelectrochemical processes.

In this poster we model the photoelectrochemical performance of boron-doped nanosstructured TiO2 film photoanodes. We focus on measures obtained by two techniques: cyclic voltammetry and transient photocurrent generated by light pulses. Our purpose is three fold: to propose a theoretical model for each phenomenon, to check its validity and to determine the values of its adjustable parameters, thus obtaining valuable qualitative and quantitative insights into the properties of the system. In particular, we use nonlinear regression techniques to compute \( \alpha^* = \arg\min_{\alpha} \sum_x |I_{\text{model}}(x; \alpha) - I_{\text{measured}}(x)|^2 \), where \( I \), \( x \) and \( \alpha \) stand respectively for the current, the independent variables and the adjustable parameters of the model.

In cyclic voltammetry, current-potential curves are generated by scanning the potential in successive oxidation and reduction directions. \( x \equiv \text{time} \in \mathbb{R} \) represents the times at which measures are taken, and \( I_{\text{model}} \) is adapted from [1]. \( \alpha \) parameters relate to capacitance and resistance components of the photoanode. The results shed light onto the structure of the electron energy levels, as well as the existence or absence of thermal equilibrium.

Transient photocurrents are measured for a series of samples throughout ten light pulses. Different series correspond to differently prepared TiO2 photoanodes. In this case, \( x \equiv (\text{sample}, \text{pulse}, \text{time}) \in \mathbb{N} \times \mathbb{N} \times \mathbb{R} \). \( I_{\text{model}} \) is a biexponential function, modelling the existence of two different processes with characteristic relaxation times \( \{\tau_1, \tau_2\} \equiv \alpha \). Our results help elucidate the influence of the film processing conditions on the obtained photocurrents.

We acknowledge financial support from Spanish MICINN/AEI (project PID2019-104272RB-C51/AEI/10.13039/501100011033), and DGA and ESF (Grupos Reconocidos T03_20R, E24_20R).
References:

P8. Other: Scientific Calculus, Approximation Theory, Discrete Mathematics
On the Motion of Two Point Masses inside a Homogeneous Cloud

Authors:
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Abstract: We consider the problem of motion of two point bodies within a homogeneous spherical cloud, under the hypotheses that the mutual interaction between these point bodies and between the bodies and the particles of the cloud is described by their Newtonian gravitational attraction. The constant density of the cloud is supposed to be sufficiently small so that the resistance of the medium to the motion can be neglected.

Under these hypotheses, the problem of relative motion of the said bodies can be recast as a perturbed Keplerian system, in which the perturbing effects are formalized by a conservative central force (RADZIEVSKIJ, [4]). With the help of the first integrals of the angular momentum and the energy, Radzievskij followed the standard solution procedure in plane polar coordinates \((r, \varphi)\), formally obtaining an orbit equation \(\varphi = \varphi(r; r_0, \varphi_0)\) in terms of an (apparently) Abelian integral.

MIHAILOVIĆ ([1]; [2]; [3]) performed different analytical treatments (vector elements, variation of constants) of this Radzievskij’s problem.

In this contributed paper we explore other analytical approaches to the solution of this problem.

References:


A subdivision scheme to refine piecewise-smooth data on triangular meshes

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Abstract:
The computational generation of curves and surfaces can be efficiently done by subdivision schemes. A well-known linear interpolatory scheme for the generation of $C^1$ surfaces is the butterfly subdivision scheme [1], which is capable of exactly reproducing third degree bivariate polynomials (when the initial data is of this kind). However, as most of the linear interpolatory schemes, it produces oscillations when the initial data has large gradients. This is a usual situation when dealing with data coming from piecewise smooth function with discontinuities.

In the univariate case, the authors in [2] explain how to transform a linear oscillatory scheme into a non-linear non-oscillatory one. The key idea is to express the given scheme as a convex combination of other schemes based on smaller stencil and, then, replace the linear averages appearing in the convex combination by non-linear analogues.

Here, we extend the ideas of [2] to the bivariate case, in particular to triangular grids, and we design a non-linear non-oscillatory version of the butterfly subdivision scheme.

This work may have applications in data compression and in the numerical solution of PDEs such as conservation laws.

References:
The Christoffel function on and in quadratic revolution surfaces

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• Miguel Piñar (mpinar@ugr.es) Universidad de Granada

Abstract:
In [1], some explicit constructions of orthogonal polynomials inside quadratic bodies of revolution, including cones, hyperboloids, and paraboloids, were given. Also, orthogonal polynomials on the surface of quadratic solids of revolution were constructed from spherical harmonics.

In both cases, the corresponding kernels can be represented in terms of the kernels associated to classical weight functions on the line. From this representation and some well known results by Totik [2], we obtain asymptotics estimates of the Christoffel functions for measures supported on and in quadratic revolution surfaces.

References:
MS01. Dynamical Systems: Theory and Applications
Central configurations of planar restricted (4+1)–body problems with some equal masses

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• Jaume Llibre (jllibre@mat.uab.cat) Universitat Autònoma de Barcelona
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Abstract:
The restricted (4+1)–body problem with the four primaries having equal masses located at the vertices of a square has exactly 13 central configurations. In this talk, we describe the evolution of the families of central configurations of the restricted (4+1)–body problem coming from the numerical continuation of these 13 central configurations when some of the masses of the primaries are decreased to zero simultaneously and the remainder ones keep constant. In particular, we consider the cases where one of the masses of the primaries decreases to zero, when either two adjacent or two opposite masses of the primaries decrease to zero simultaneously, and when three masses of the primaries decrease to zero simultaneously. In all these cases, each one of the 13 families of central configurations starting at a central configurations of the square restricted (4+1)–body problem ends either at a central configuration of the restricted (4+1)–body problem corresponding to a bifurcation value of the mass parameter or to a central configuration of the corresponding restricted (n + m)–body problem with n + m = 5 and n, m = 1, 2, 3, 4.
Arnold Diffusion in the Restricted Planar Elliptic Three Body Problem

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• Tere Seara (tere.m-seara@upc.edu) Universitat Politecnica de Catalunya

Abstract: Consider the RPE3BP with any mass ratio $\mu \in (0, 1/2)$ and any eccentricity $\epsilon \in (0, 1)$. We build orbits along which the angular momentum of the massless body becomes unbounded as $t \to \infty$. The construction relies on an Arnold diffusion mechanism by designing a transition chain of periodic orbits. For that, we identify an invariant manifold at infinity and prove that its four dimensional stable and unstable manifolds intersect transversally along two different homoclinic manifolds. These homoclinic manifolds define two scattering maps which encode the dynamics along the heteroclinic orbits. We prove that they can be combined to build a sequence of periodic orbits connected by heteroclinics along which the angular momentum grows unboundedly. One of the main difficulties is that the splitting angle between the stable and unstable manifolds is exponentially small with respect to the angular momentum and therefore Melnikov theory cannot be applied. This is joint work with Marcel Guardia and Tere Seara.
Dynamics of a non-symmetric dipole-segment asteroid model

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• Patricia Sánchez-Martín (patricias@unizar.es) Centro Universitario de la Defensa, Zaragoza.

Abstract: Understanding the gravitatory dynamics around asteroids is crucial for approaching and orbiting close to them. Because of their small size and irregular shape, the description of their gravitational field is challenging. Representations of their potential include Legendre polynomial series, polyhedral and mascon models, as well as other simplified models. A simplified mathematical model provides qualitative information of the environment of the asteroid that is useful for predicting the orbital motion of particles, therefore for asteroid observation mission planning. In particular, this work is derived from previous researches for modelling an elongated asteroid by a massive finite segment potential [1] and more recently by a dipole-segment model [2].

In the present work we explore the dynamics of the dipole-segment model with unequal endpoint masses. In particular, we study the stable and unstable invariant manifolds associated to the unstable equilibrium points. These manifolds allow the comprehension of the dynamical problem far from the equilibrium point [3], acting as tubes which transport matter to and from the neighbourhood of the equilibrium point using homoclinic and heteroclinic connections [4]. In this work we focus on the evolution of heteroclinic connections depending on the parameter that rules the asymmetry of the end masses in the dipole-segment.

References:
Semi-analytical computation of heteroclinic connections between center manifolds with the parameterization method

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- Alex Haro (alex@maia.ub.es) Centre de Recerca Matemàtica & Departament de Matemàtiques i Informàtica, Universitat de Barcelona
- Josep-Maria Mondelo (jmm@mat.uab.cat) CERES-IEEC & Departament de Matemàtiques, Universitat Autònoma de Barcelona

Abstract: This talk is devoted to present a methodology for the computation of heteroclinic connections between invariant tori in Hamiltonian systems using the parameterization method for the computation of invariant manifolds of fixed points of flows. We introduce a new parameterization style that uncouples the hyperbolic part from the central one in the system of reduced equations, providing parameter space with fiber structure. The method is applied to compute approximations of heteroclinic orbits between invariant tori around librational equilibrium points of the spatial, circular, Restricted Three Body Problem, which are then refined using a Newton-like method. Particularly, for the Earth-Moon mass parameter, we provide representations of the whole set of heteroclinic connections between iso-energetic slices of the center manifold of $L_2$ and $L_1$ with minimum number of Moon revolutions, for representative values of the energy.

References:

Invariant tori around the Moon including the solar gravitational effect

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Abstract:
The Bicircular Problem (BCP) is a time-periodic perturbation of the well-known Restricted Three-Body Problem (RTBP), that introduces the gravitational effect of a third massive body, [2]. In our case, we consider the planar Earth-Moon RTBP subjected to the gravitational field of the Sun, described by a non-autonomous Hamiltonian:

\[ H = \frac{1}{2}(p_x^2 + p_y^2) + yp_x - xpy - \frac{1 - \mu}{r_{PE}} - \frac{\mu}{r_{PM}} - \frac{m_s}{r_{PS}} - \frac{m_s}{a_s^2}(y\sin(\omega_s t) - x\cos(\omega_s t)), \]

where \( r_{PE}, r_{PM}, \) and \( r_{PS} \) are the distances to the particle from the Earth, the Moon and the Sun, respectively, \( \mu = 0.012150582 \) is the Earth-Moon mass parameter, \( m_s \) is the solar mass, \( a_s \) is the distance between the Sun and the Earth-Moon barycenter and \( \omega_s \) is the frequency effect of the Sun.

In the RTBP there are many families of periodic orbits. In particular, there is a family of retrograde periodic orbits around the Moon, contained in the horizontal plane, usually called Family C of retrograde periodic orbits, [1]. This family is stable, what makes it suitable for many purposes like performing physical measurements or just to be employed as lunar parking orbits where to capture an asteroid or to keep materials for some mission in the Moon. In the BCP, these periodic orbits become 2D invariant tori, under generic non-resonance and non-degeneracy conditions are satisfied.

We compute the invariant tori making use of a temporal Poincaré map, \( P \), in which they are seen as one-dimensional invariant curves. In this process, we find difficulties due to the closeness of these quasi-periodic solutions to a resonance with the natural frequency of the problem, what gives rise to the appearance of chaos. To show this chaos, we analyse the Lyapunov exponent in a neighbourhood of these quasi-periodic solutions: we take a point in one of the invariant curves of the map \( P \), define a mesh of initial conditions around it and colour them according to their Lyapunov exponent.

References:
A dynamical definition of the Sphere of Influence of the Earth

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Abstract:
Planetary close encounters can drastically modify the trajectories of small bodies, such as asteroids, comets or spacecrafts [1]. The phenomenon is significant in different contexts, from the problem of impact monitoring to the design of interplanetary trajectories. One of the classical methods used to model close encounters is the patched-conics approximation, consisting in patching two-body solutions at the boundary of the so-called planetary sphere of influence [2]. The concept of sphere of influence is still ambiguous as different definitions exist, typically based on the relative position between the planet and the body of interest [3]. We propose a new definition of the sphere of influence which depends on both the relative position and velocity of the small body with respect to the planet. Our study is focused on the Earth and relies on an optimization process, in which we compare the patched-conics orbit and the trajectory obtained in the framework of the circular restricted three-body problem. During the presentation, we will describe the optimization process implemented and we will discuss the results.

References:

Chaotic coorbital motions to $L_3$ in the Restricted Planar Circular 3-Body Problem

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Abstract:

The Restricted 3-Body Problem models the motion of a body of negligible mass under the gravitational influence of two massive bodies, called the primaries. If the primaries perform circular motions and the massless body is coplanar with them, one has the Restricted Planar Circular 3-Body Problem (RPC3BP). In synodic coordinates, it is a two degrees of freedom Hamiltonian system with five critical points, $L_1, \ldots, L_5$, called the Lagrange points. The Lagrange point $L_3$ is a saddle-center critical point (collinear with the primaries and beyond the largest one) with a 1-dimensional stable and unstable manifold. When the ratio between the masses of the primaries $\mu$ is small, the modulus of the hyperbolic eigenvalues are weaker, by a factor of order $\sqrt{\mu}$, than the elliptic ones.

In this work, we present an asymptotic formula for the distance between the stable and unstable manifolds of $L_3$. Due to the rapidly rotating dynamics, this distance is exponentially small with respect to $\sqrt{\mu}$ and, as a result, classical perturbative methods (i.e the Melnikov-Poincaré method) can not be applied. By means of this result, we prove that the stable and unstable invariant manifolds of Lyapunov periodic orbits exponentially close to $L_3$ intersect transversally. By the Smale-Birkhoff homoclinic theorem, this implies the existence of chaotic motions exponentially close to $L_3$ and its invariant manifolds.

References:


Splitting and coexistence of any number of strange attractors in families of expanding baker maps

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Abstract: We consider a two-parameter family $\Gamma_{a,\theta}$ of expanding baker maps on the plane, being $a > 1$ and $0 < \theta < \pi$ an expansion rate and a rotation angle, respectively. If $\theta/\pi$ is a rational number, we prove that $\Gamma_{a,\theta}$ exhibits strange attractors for every $a$ sufficiently close to 1. We also study how such attractors may split into other ones of a larger number of connected pieces as $a$ decreases to 1.
Splitting of separatrices for rapidly forced pendulum with a perturbation without first harmonic

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Abstract:
When an equilibrium point in a planar integrable Hamiltonian system has coincident stable and unstable invariant manifolds—a separatrix—applying a perturbation can lead to its splitting. A usual measure of the splitting for periodic perturbations is the distance, $S(x)$, between stable ($W^s$) and unstable manifolds ($W^u$), of the associated stroboscopic map:

In this work we study the splitting distance of a rapidly perturbed pendulum $H(x, y, t) = \frac{1}{2} y^2 + (\cos(x) - 1) + \mu (\cos(x) - 1) g(\frac{\tau}{\varepsilon})$ with $g(\tau) = \sum_{|k|>1} g[k] e^{ik\tau}$ a 2\pi-periodic function and $\mu, \varepsilon \ll 1$. Systems of this kind undergo exponentially small splitting and when $\mu \ll 1$ it is known that the Melnikov function actually gives an asymptotic expression for the splitting function provided $g[\pm 1] \neq 0$. Our study focuses on the case $g[\pm 1] = 0$ and it is motivated by two main reasons. On the one hand the general understanding of the splitting, as current results fail for a perturbation as simple as $g(\tau) = \cos(5\tau) + \cos(4\tau) + \cos(3\tau)$. On the other hand, a study of the splitting of invariant manifolds of tori of rational frequency $p/q$ in Arnold’s original model for diffusion leads to the consideration of pendulum-like Hamiltonians with $g(\tau) = \sin\left(p \cdot \frac{\tau}{\varepsilon}\right) + \cos\left(q \cdot \frac{\tau}{\varepsilon}\right)$, where, for most $p, q \in \mathbb{Z}$ the perturbation satisfies $g[\pm 1] \neq 0$.

To tackle the problem we use a splitting formula based on the solutions of the inner equation and make use of the Hamilton-Jacobi formalism. In our main result we show that the leading exponentially small term appears at order $\mu^n$, where $n$ is an integer determined exclusively by the harmonics of the perturbation. As was expected, the Melnikov function is in fact not a correct approximation for the splitting in this case.
Scattering in the elastic collision of He atoms off a corrugated Cu surface

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Abstract:
We study numerically the scattering of He atoms off corrugated Cu surfaces (He-Cu model in short). The model is conservative and can be described by the Hamiltonian

\[ H(x, z, p_x, p_z) = \frac{1}{2m}(p_x^2 + p_z^2) + V(x, z), \]

where \( x, z \) are the parallel and perpendicular directions to the surface, respectively, \( p_x, p_z \) are the corresponding momenta, and \( V(x, z) \) is the Morse potential that describes the interaction between the incident atom with the surface that depends on its corrugation:

\[ V(x, z) = De^{-\alpha z}(e^{-\alpha z} - 2) + De^{-2\alpha z} \sum_{n \geq 1} r_n \cos \left( \frac{2\pi nx}{a} \right) \pm s_n \sin \left( \frac{2\pi nx}{a} \right), \]

where \( a \) is the Cu unit cell length ([4]). The coefficients \( D, \alpha, r_n \) and \( s_n \) are determined experimentally.

The aim is to prove the existence of oscillatory motions, that is, solutions of the system in which the helium atom reach higher and higher distances from the copper surface, while it always return to a finite distance:

\[ \lim_{t \to \infty} \sup z(t) = \infty, \quad \lim_{t \to \infty} \inf z(t) = \text{finit}. \]

We will show numerical evidences supporting this statement, that is, the existence of a homoclinic connection to a parabolic periodic orbit at infinity. First, we use the parametrization method adapted to the He-Cu model to obtain an approximation of any order of the invariant manifolds associated to the parabolic periodic orbit. Second, we compute the splitting angle at the intersection of the invariant manifolds, which is exponentially small.

References:

Ejection/collision orbits and beyond

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Abstract:
We will discuss the existence, properties and dynamical consequences of ejection/collision orbits in different n-body problems.
Ejection/collision orbits for the hydrogen atom in a circularly polarized microwave field

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Abstract:
We study the ejection/collision orbits of the problem of the Hydrogen atom interacting with a circularly polarized microwave field. This problem can be modelled as a perturbed Kepler problem (see [1]) and its Hamiltonian formulation is given by

\[ H = \frac{1}{2}(p_x^2 + p_y^2 + p_z^2) - \frac{1}{r} + F(x \cos \omega t + y \sin \omega t), \]

where \((x, y, z)\) and \((p_x, p_y, p_z)\) are the canonical coordinates and their conjugate momenta, \(r^2 = x^2 + y^2 + z^2\), \(\omega\) is the angular frequency of the microwave field, \(t\) is the time and \(F > 0\) is the field strength. After a suitable change of coordinates and time we can express the problem with a Hamiltonian with two degrees of freedom, that depends on a single parameter \(K > 0\) and is expressed as a simple perturbation of the well known Kepler problem. Despite its simplicity, this problem has a very complex dynamic (see [2]).

In this talk, we will restrict to the planar case and using the Levi-Civita regularization we will focus on the so called \(n\)-ejection-collision orbits (\(n\)-ECOs). These orbits eject from the nucleus (located at the origin), describe \(n\) maximum distance passages from the origin and finally collide with it. On the analytical side, we claim and prove the existence of two families of \(n\)-EC orbits for big enough ranges of the energy following the ideas introduce in [3] and numerically we will study the evolution of these families.

References:
Resonances in the hydrogen atom

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Abstract: In this talk we discuss the dynamics of an electron about an hydrogen atom under the excitation of a laser field [1, 3, 2]. This situation can be modelled with by the following two-degrees of freedom Hamiltonian system depending on a parameter $K$.

$$H = \frac{1}{2} (p_x^2 + p_y^2) - x \cdot p_y + y \cdot p_x - \frac{1}{\sqrt{x^2 + y^2}} + K \cdot x.$$  

Moreover, it can be regarded as a perturbation of the Kepler problem. For the non-perturbed system, the dynamics is integrable and, generically, the solutions are confined in invariant tori carrying quasi-periodic motion. When the perturbation is active (i.e. the effect of the laser is considered, $K \neq 0$), the system ceases to be integrable. Moreover, it has two equilibria ($L_1$, which is unstable and $L_2$). We focus on $L_2$ which is a center provided that the parameter is below some critical value. Therefore, it has associated regions of stability that corresponds to bounded motions of the electron about the core. We are interested in studying how these stability regions get destroyed by the act of low-order resonances. The destruction of these stability regions give rise to trajectories that abandon the core after a long time.

References:


Metapopulation interaction models for infectious diseases

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Abstract:
In this talk we discuss the evolution of a contagious illness though a SIR model. As a novelty, the model separates the population in different patches which has its own individual interaction. This is intended to avoid the mass action mixing assumption, that is, the assumption that each individual can interact with all of the other individuals of the population. The interaction between the different individuals (Susceptible, Infected, Removed, etc) can be modelled by means of a connectivity matrix. Notice that this models can provide dynamical insight on, for instance, the effect of confining the infected class.

We tackle two different problems:

1. How the existence of two different patches affect the epidemic threshold theorem.
2. How the existence of a large number of patches affect the evolution of the epidemic.
On the persistence of the tripod gait

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Abstract: The study of the synchronization patterns in biological processes is a growing discipline. Small networks of neurons model central pattern generators (CPG) that control insect locomotion (see [1],[2]).

Firstly, we study small CPGs (6-neuron model) for insect locomotion where each neuron follows the Hodgkin-Huxley like model of [2], presenting a roadmap with exhaustive information on the dynamical behavior of a single neuron [3], using Spike-counting diagrams and bifurcation analysis.

Then, we analyze the complete system, performing automatic detection techniques combined with quasi-Monte-Carlo sweeping methodologies and continuation techniques. These methods allow us to obtain a complete picture of pattern evolution on the movement gaits of the CPG leading to a global dominance of the tripod gait on the fast movement regime (see [3],[4]).

Finally we explore other CPGs with similar behaviour.

References:
Optimal control of neural populations for Communication Through Coherence

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Abstract: In this talk, I will show how to synchronize populations of neurons in order to allow them to communicate. On many scales in the brain, from single neurons to populations of tens of thousands and more, oscillatory behaviors can be scouted. We are interested in Excitatory-Inhibitory populations of neurons which are involved in a vast number of cognitive tasks, among which perception and attention through the well established Communication Through Coherence (CTC, [2]) mechanism. CTC theorizes that populations of neurons must synchronize in order to communicate between themselves, giving a structural approach to medium-scale functioning of the brain. The dynamics of our populations are modeled through a mean field approach due to Montbrió and al [3], and we aim at synchronizing populations thanks to control theory where the control itself is the action of a higher cortical area (the so-called topdown mechanism).

Two populations (oscillators) will perturb a third system (receiver), and we want this third population to synchronize with, let us say, the first oscillator. Let $F$ be a vector field in $\mathbb{R}^n$ with a stable limit cycle $\gamma$ (the solution and the cycle itself will (abusively) be denoted the same).

\[
\begin{align*}
\dot{\theta}_1 &= \frac{1}{T_1} \\
\dot{\theta}_2 &= \frac{1}{T_2} \\
\dot{x} &= F(x) + \epsilon S(\theta_1, \theta_2) + U(t)
\end{align*}
\]

$S$ here models the perturbation of the two populations. The control $U = (u, 0, \ldots, 0)^T$ is a modulation of the voltage coordinate only, thus our system will not be globally controllable, since we are only interested in synchronizations, i.e., controlling the period of the oscillations, we use the well known Phase-Amplitude reduction [1] of a limit cycle:

\[
\begin{align*}
\dot{\theta} &= \frac{1}{T} + \nabla_x \theta.(U(t) + \epsilon S(\theta_1, \theta_2)) \\
\dot{\sigma} &= \Lambda \sigma + \nabla_x \sigma.(U(t) + \epsilon S(\theta_1, \theta_2)).
\end{align*}
\]

where $\theta$ is the extended period, and $\sigma$ the amplitude - and make full use of the recent developments in this area in order to find the best control strategy (regarding some cost function that I will specify during the talk).

References:
Coriolis coupling in a Hénon-Heiles system

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Abstract: We study the impact of a Coriolis term in the dynamics of a perturbed Hénon-Heiles Hamiltonian. The strength of the Coriolis coupling is measured by a frequency $\omega$ that regulates two different regimes, depending on whether or not $|\omega|$ is greater than one. We restrict our study to $\omega \in [0, 1)$, where richer dynamics appears. Poincaré surfaces of section are used to show how the strength of the Coriolis coupling controls the size of the trapping area. While, for $\omega = 0$, most of the orbits escape, for $\omega \approx 1$ most of the orbits remain trapped. The transition from one situation to the other one reveals complex resonant structures giving rise to a chaotic sticky region of long living orbits. The study of the escape basins reveals that they evolve from a complex structure, with fractal boundaries, to basins with smooth boundaries. Explicit computation of the evolution of the basin entropy confirms this fact. The escape probability as a function of $\omega$ is also calculated. Both the evolution of the escape probability and the entropy are not monotonic and exhibit intricate and complex dynamics for intermediate values of $\omega$. 
Computable normal forms for piecewise smooth systems with a pseudo-focus

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Abstract: For planar systems having a pseudo-equilibrium point of focus type within its discontinuity line, suitable normal forms are introduced. Here, classical theory of normal forms is adapted for dealing with piecewise smooth systems with a common invisible tangency from each side. The methodology looks for eliminating non-essential terms in the expression of the vector field, and is based on expanding such a vector field as a sum of quasi-homogeneous terms and then, applying adequate changes of variables that preserve every point of the discontinuity line. Once obtained these canonical forms, it is easier to compute the associated half-return maps and to determine the maximal number of periodic orbits than can bifurcate from a pseudo-focus. A recent conjecture about the behaviour of the displacement function around an invisible fold-fold singularity is shown to be true. Furthermore, we illustrate the obtained results by considering a rather general family of linear-quadratic planar systems.

References:

Dynamic Hopf-like bifurcation in piecewise linear systems

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Abstract: The slow-passage phenomenon is characterized in slow-fast differential systems as a time-delay in changing behavior presented by the corresponding orbit when it crosses a bifurcation point near a slow manifold. This phenomenon is widely known in the field of dynamical systems and it is used to understand some qualitative and quantitative aspects appearing in some slow-fast differential systems, such as complex oscillations. Since piecewise linear (PWL) systems have proven to be useful to provide salient features of smooth systems while being more amenable to theoretical and computational analysis, in this work, we aim to see for the first time that the slow-passage phenomena can be also observed in PWL systems. In particular, we show under which conditions, in terms of number of linear regions, a PWL slow-fast system is able to reproduce this phenomenon through a Hopf-like bifurcation.

References:

Limit cycles of polynomially integrable discontinuous piecewise linear differential systems

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Abstract: We try to answer how many algebraic limit cycles can exhibit the discontinuous piecewise linear differential systems separated by a line when the two linear differential systems have polynomial first integrals. It should be distinguished the case with at least one of the systems being Hamiltonian. Under this assumption and an straight line as a boundary these piecewise differential systems at most have one limit cycle, [1]. Meanwhile purely Hamiltonian case with a blended boundary can exhibit even more. Our study is based on a complete characterization of the linear differential systems having polynomial first integrals.

References:

Limit cycle bifurcation from infinity in 3D relay systems

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Abstract: We characterize a limit cycle bifurcation from infinity in 3D relay systems, which belong to the class of three-dimensional symmetric discontinuous piecewise linear systems with two zones. More precisely, we consider canonical relay systems in the form

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix} = \begin{pmatrix}
2\sigma + \lambda & -1 & 0 \\
-1 & 2\sigma\lambda + \sigma^2 + \omega^2 & 0 \\
0 & -1 & \lambda(\sigma^2 + \omega^2)
\end{pmatrix} \begin{pmatrix}
x \\
y \\
z
\end{pmatrix} - \text{sign}(x) \begin{pmatrix}
b_1 \\
b_2 \\
b_3
\end{pmatrix},
\]  

where the vector \((x, y, z)^T \in \mathbb{R}^3\) stands for the state variables, the dot represents derivative with respect to the time, the vector \((b_1, b_2, b_3)^T \in \mathbb{R}^3\) is constant, and the matrix ruling the dynamics has eigenvalues \(\lambda\) and \(\sigma \pm i\omega\), where \(\omega > 0\). The ambiguity in the definition of the vector field at the plane \(x = 0\) will be not relevant as long as we only consider the crossing dynamics.

As done in [1], the closing equations method is adapted to justify the existence of a great amplitude limit cycle that bifurcates from infinity and to provide analytical expressions for its amplitude, period and stability. Our first result assures that if the non-degeneracy parameter \(\delta = b_3 - b_2\lambda - b_1\omega^2\) does not vanish, then for \(\sigma = 0\) system (1) undergoes a Hopf bifurcation from infinity; more precisely, one symmetric limit cycle of great amplitude appears for \(\delta\sigma < 0\) and \(\sigma\) sufficiently small. In particular, when \(\lambda \neq 0\), if \(\delta > 0\) and \(\lambda < 0\), then the limit cycle bifurcating for \(\sigma < 0\) is orbitally asymptotically stable. Otherwise, if \(\delta < 0\) or \(\lambda > 0\) the bifurcating limit cycle is unstable, being completely unstable when both inequalities hold.

The co-dimension two bifurcation appearing for \(\delta = 0\) is also analyzed. These results are exploited to complement the work presented in [2].

References:


Zero-Hopf bifurcation at infinity in 3D piecewise linear systems with symmetry

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Abstract: The zero-Hopf bifurcation for differentiable systems has been widely studied, see for example [2]. On the other hand, in the frame of piecewise linear differential systems, this bifurcation has been scarcely analyzed, see [3], where the zero-Hopf bifurcation of a symmetric piecewise linear differential system with three zones in \( \mathbb{R}^3 \) was characterized obtaining the existence and stability of three periodic orbits bifurcating from the origin and two non trivial equilibria. These equilibrium points must be of the zero-Hopf type, that is, with eigenvalues 0 and \( 0 \pm \omega \), for the zero-Hopf bifurcation to take place.

In the present work, the family of systems studied has the complex eigenvalues \( \rho \varepsilon \pm i \omega \) and the real one \( -\varepsilon \) and the non trivial equilibrium points tend to infinity when \( \varepsilon \) tends to zero, converting them in zero-Hopf equilibria. These zero-Hopf equilibria give place to a large amplitude bifurcating limit cycle, increasing the catalog of bifurcations at infinity in piecewise linear systems, see [1].

Theoretical results determining period, amplitude and stability of the large amplitude bifurcating limit cycle are provided.

These results are applied to a Bonhöfer-van der Pol electronic oscillator.

References:


Birth, transition and maturation of canard cycles in PWL systems

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Abstract: In this talk we deal with the canard regime as a part of the canard explosion [1, 2]. In particular, we explore the canard explosion taking place in a PWL version of the van der Pol equation having a flat critical manifold. The proposed analysis involves the identification of two specific canard cycles, one at the beginning and the other at the end of the canard regime, here called birth and maturation of canards, respectively. Moreover, inside the canard regime, we also analyse the transition from small amplitude canard cycles (canards without head) to large amplitude canard cycles (canards with head) by identifying the maximal canard, the transitory canard, and the maximum period canard. Moreover, we then prove that all these cycles are, in fact, different dynamical objects, and we order them by amplitude size.

There have been several works in the classical framework addressing the transitory regime, but from a numerical point of view [3]. Some of these works involve systems exhibiting a flat slow manifold. This is a good choice as a first approximation to the problem, because the flatness of the critical manifold allows an interval of the amplitude of the canard cycles to become non-bounded as $\varepsilon$ tends to zero. Thus, regarding to the amplitude of the cycles, canards appear further apart each to other, allowing for a deep analysis of the transitional region going from the headless canard cycles to the canard cycles with head. For that reason we have considered a four-zonal PWL system in which the critical manifold in the lateral left linear region is flat.

References:


The Hindmarsh-Rose model: classification of spike-adding processes and bifurcation structure

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Abstract:
Mathematical neuroscience has a prominent role nowadays. Understanding a system as complex as the brain requires to achieve a deeper understanding of the dynamics of neurons, its basic units. A relevant behavior displayed by different types of neurons is bursting, which consists of the alternation between periods of spiking with periods of quiescence. When a bursting neuron goes from producing \( n \) spikes per burst to producing \( n + 1 \), we talk about the process of spike-adding. We studied the different spike-adding processes that can be found in the Hindmarsh-Rose neuron model, which is widely accepted as an important prototype for fold/hom and fold/Hopf burstings. There are two different mechanisms for the fold/hom case (continuous and chaotic) and a single (and continuous) mechanism for the fold/Hopf case. While all these processes are presented, to some extent, in the literature, our study gives a unified vision that allows to construct a spike-adding map determined by the bifurcation structure. Moreover, our results allow to understand the dynamics exhibited when borders are crossed, a crucial point not previously studied.

References:
Analysis of travelling waves in cortical spreading depression models involving multiple time scales

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Abstract: Cortical spreading depression (CSD) is a self-propagating wave of neuronal depolarization (saturation of activity) in the cortex, implicated in many brain disorders such as migraine and ischemia. CSD waves are characterized by a disruption of transmembrane ionic gradients, the collapse of ion homeostasis, cell swelling, and elevated extracellular $K^+$ levels. Astrocytes take part in regulating $K^+$ concentration by soaking up its accumulation from the extracellular space, thus possibly delaying, or even preventing, the initiation of these CSD waves.

In this talk we consider a reduced version of a biophysical model developed by Huguet et al (2016) [1] for the instigation and spreading of CSD waves, consisting of reaction-diffusion equations. The reduced model (Lee [2]) considers only the neuronal and astrocytic membrane potentials and the extracellular $K^+$, capturing only the instigation process implicated in such waves. We perform a study of the model using computational and analytical tools to provide semi-analytical results on the existence of a travelling wave solution (front) in the model and its velocity as a function of certain biologically relevant parameters. Our approach consists in looking for a heteroclinic connection in a system of ordinary differential equations with a slow-fast dynamics. We tackle this problem by, exploiting the different time scales, dissecting the system to the fast and slow subsystems, to construct a heteroclinic solution in the singular limit that persists away from it. Our results shed light into the mechanisms responsible for the initiation of these waves, that we plan to extend to the full model in the future.

References:
Ordered structures of chaotic invariant sets on square-wave neuron models

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Abstract: The present work focuses on square-wave bursting neuron models. We analyze, joining different techniques and connecting symbolic dynamics with recent results on the bifurcation analysis, the evolution of the chaotic invariant set along the three-parameter region where the chaotic spike-adding occurs [1]. The basic ingredient necessary to carry out our study is to have a region of the phase space in which there is a chaotic invariant set whose FRM is unimodal [2]. This situation occurs both in the Hindmarsh-Rose neuron model and in a realistic leech heart neuron model [3] although other neuron models seem to obey the same structure.

We will show how the simple grammar defined with two symbols from the unimodal FRM of the chaotic invariant allows to assign symbolic sequences to all the orbits of the system. These symbolic sequences make it possible to define an order and a sequence in the appearance and disappearance of the periodic orbits. This ordering in the parametric space, and the region of the phase space that the chaotic set occupies, also condition the position of the periodic orbits that are generated at any parametric conditions. This influence is maintained throughout the entire parametric region in which such family of periodic orbit exists. In summary, we will illustrate how the symbolic analysis of the existing orbits in the dynamics of the system establishes an order in the chaos that underlies said dynamics.

References:

The Poincaré map of degenerate monodromic singularities with Puiseux inverse integrating factor

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Abstract: We consider the center-focus problem for analytic families of planar vector fields having a monodromic singularity with characteristic directions when the parameters lie in a set Λ.

First we give a method to compute the center conditions of the family provided there is an inverse integrating factor whose expression in polar coordinates has the exceptional divisor of the first polar blow-up as isolated zero-set. These results are based on the work [1].

Secondly we characterize the structure of the asymptotic Dulac series of the Poincaré map when the family possesses a Puiseux inverse integrating factor in terms of its multiplicity and index. This characterization is only valid in a restricted monodromic parameter space Λ\Λ* associated to the non-existence of local curves with zero angular speed. As a by product we are able to study the center-focus problem (under the assumption of the existence of some Cauchy principal values) in very degenerated cases where no other tools are available. This point of view is based on the paper [2].

Some non-trivial examples are analyzed.

References:


Invariant algebraic curves for certain generalized Liénard differential system via Puiseux integrability

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Abstract: We solve the problem of finding the invariant algebraic curves of a generalized Liénard differential system \( \dot{x} = y, \dot{y} = -f(x)y - g(x) \), where \( \deg f = m \) and \( \deg g = n \) and with \( n = m + 1 \), generalizing the known previous examples presented. In particular it is studied the case \( m = 3 \) and \( n = 4 \). In order to bound the degree of such invariant algebraic curves we use the Puiseux integrability based on finding the structure of Puiseux series that are solutions of the first order ordinary differential equation associated to the generalized Liénard differential system. The relation between Puiseux series and irreducible invariant algebraic curves is described in [1, 2, 3]. The difficulties in applying the Puiseux integrability are shown even when the degrees of the invariant curves are bounded, see [4].

References:
Period function of planar turning points

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Abstract: In this talk we settle the background for the conjecture about a uniform upper bound for the number of critical periods inside classical Liénard systems of fixed degree, formulated by De Maesschalck and Dumortier in [1]. Using singular perturbation theory and the family blow-up we study the period function of planar generic and non-generic turning points, solving part of the conjecture [2]. In the generic case (resp. non-generic) a non-degenerate (resp. degenerate) center disappears in the limit $\epsilon \to 0$, where $\epsilon \geq 0$ is the singular perturbation parameter. We show that, for each $\epsilon > 0$ and $\epsilon \sim 0$, the period function is monotonously increasing (resp. has exactly one minimum). The result is valid in an $\epsilon$-uniform neighborhood of the turning points.

References:
Dissecting bifurcation diagrams: geometric bifurcations

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Abstract: Exploring a $p$-dimensional parameter space through $(p-1)$-dimensional slices, one can detect changes that are only determined by the geometry of the manifolds that compose the bifurcation diagram. These changes can be referred as bifurcations, but they must be understood in a geometrical sense, since they reflect variations in the topology of the intersections between the slices and the bifurcation —in a dynamical sense— set. Although the notion is new, geometric bifurcations have already been partially discussed in the literature, see for instance [1, 5]. To provide a theoretical support for this whole scenario we use the framework of singularities theory for differentiable mappings and, particularly, the Morse Theory. To illustrate the notion of geometrical bifurcation we will see how, working within a 3D parameter space, they arise in the context of two models of neuron activity: the Hindmarsh-Rose model [2, 3] and the FitzHugh-Nagumo system [4]. Both are fast-slow systems which include a small parameter that controls the time scale of a slow variable. Geometrical bifurcations are observed on slices corresponding to fixed values of this distinguished small parameter.

References:

Lie point symmetry algebras of some quadratic differential systems: dynamical consequences

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Abstract: We consider the full family of quadratic differential system on the plane possessing a non-degenerate monodromic singularity, that is, a non-degenerate center or focus. Under some restriction we transform the system into a scalar autonomous second–order differential equation and next we characterize its Lie point symmetry algebras, see [2] for example. Based on this classification we can analyze several dynamical consequences such as the existence of invariant (non–autonomous first integrals) or the analysis of periodic orbits. This work is inspired on the ideas of [1].

References:


Bifurcations in the Riemann ellipsoids

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Abstract:
We study steady motions of an ideal incompressible homogeneous self-gravitating fluid mass. The ones that retain an ellipsoidal shape are called Riemann ellipsoids.

Researchers as distinguished as Newton, MacLaurin, Dirichlet or Riemann have paid attention to this problem. More recently, Chandrasekhar [1] has used Riemann ellipsoids to model and analyze the stability of rotating stars.

Studies on the nonlinear stability of these ellipsoids have been carried out by Fassó and Lewis [2], for instance. Up to our knowledge, there has not been an analytical study of the bifurcations occurring in this problem.

We present an approach based on Hamiltonian normal forms to prove the existence of two types of bifurcations: steady-state and Hamiltonian Hopf [3]. Using symplectic reduction, the problem is formulated as a Hamiltonian system of four degrees of freedom. These bifurcations can be understood as transitions from stable to unstable regimes of the ellipsoids’ configurations.

References:
Fast-slow analysis in cardiac dynamics

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Abstract: In this communication, we present the dynamical mechanisms underlying the generation of arrhythmogenic early afterdepolarizations (EADs) in a three-variable model of a mammalian ventricular cell. This model is analysed by considering a 1-fast, 2-slow variable decomposition of the system describing the cellular action potential. We use sweeping techniques, such as the spike-counting method, and bifurcation and continuation methods to identify parametric regions with EADs. We show the existence of isolas of periodic orbits organizing the different EAD patterns and we provide a preliminary classification of our fast-slow decomposition according to the involved dynamical phenomena.

References:

MS02. Numerical approximation of hyperbolic PDE systems and their applications
Minimally implicit methods for the resolution of the neutrino transport equations

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Abstract: The evolution of many astrophysical systems is dominated by the interaction between matter and radiation such as photons or neutrinos. The dynamics can be described by the evolution equations of radiation hydrodynamics in which reactions between matter particles and radiation quanta couples the hydrodynamic equations to those of radiative transfer [1, 2]. The numerical treatment has to account for their potential stiffness (e.g., in optically thick environments). In this talk, we will present a new method to numerically integrate these equations in a stable way by using minimally implicit Runge-Kutta methods. With these methods, the inversion of the implicit operator can be done analytically. We also take into account the physical behavior of the evolved variables in the limit of the stiff regime. We will show the results of applying this method to the reactions between neutrinos and matter in core-collapse supernovae simulations.

References:


High order numerical schemes for generalized models in chromatography

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Abstract:
The equilibrium dispersive model for liquid chromatographic processes consists in a system of nonlinear convection-diffusion partial differential equations for the concentration of each injected liquid component.

There are several difficulties posed in the numerical approximation of the solution of these equations: Firstly, the fluxes depend on the concentration of the liquid (mobile) phases, but the conserved variables are the concentration of each substance, which consists of a fraction of mobile phase and a complementary fraction of stationary phase, in which the concentration of stationary phase adsorbing each injected substance is given by nonlinear relations, the adsorption isotherms; Secondly, when the apparent axial dispersion coefficient (see [4]), i.e., the diffusion coefficient, is small but non-negligible, strong gradients are inevitably developed and the numerical schemes should cope with them by being based on conservative discretizations, which require the functional inversion of the nonlinear relationship between primitive variables (the variables the fluxes depend on) and conserved variables (see [2]); Thirdly, an explicit discretization of the diffusive fluxes may impose a stability restriction on the time step that might compromise efficiency.

In [2] Langmuir isotherms are used and the latter issues are tackled through an efficient functional inversion for getting primitive variables from conserved ones and Implicit-Explicit time-discretizations (see [1]), in which the diffusion terms are treated implicitly and the convective terms explicitly.

The burden of these techniques is that the convective fluxes are obtained through a high-order reconstruction of the components of split physical fluxes through the Global Lax-Friedrichs flux-splitting. The use of this flux-splitting (or an upwind flux-splitting) in the componentwise setting often yields spurious oscillations near discontinuities (see [3]).

In this paper we extend the techniques introduced in [2] to generalized Langmuir isotherms (Tóth isotherms) and propose a characteristic-based upwind numerical flux (see [3]) that yields oscillation-free results. Some experiments are performed to show the relative efficiency gains of the proposal.

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References:

Integrating functions with singularities: adapted quadrature rules

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Abstract: In the numerical simulation of physical phenomena modeled through differential equations, discontinuities can be present in some way and thus they constitute an important challenge in the design of accurate numerical methods. Moreover, in some problems, the position of the discontinuities might be known and the jump in the solution and its derivatives might be available due to the particular characteristics of the physical problem. In this case, it might be interesting to use this information to obtain accurate numerical quadrature formulas that allow approximating the integral of the discretized solution of the differential problem over a certain interval. This work is devoted to the construction and analysis of a new nonlinear technique that allows improving the accuracy of classical numerical integration formulas of any order when dealing with data that contains singularities and when the function that is to be integrated is only known at grid points. The novelty of the technique consists in the inclusion of correction terms with a closed expression that depends on the size of the jumps of the function and its derivatives at the singularities, that are supposed to be known. The numerical experiments performed allow us to confirm the theoretical conclusions reached in this paper.

References:
Well-balanced implicit-explicit Lagrange-Projection scheme
for the one-dimensional shallow water system

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Abstract:
In this work we present first and second order numerical approximations of the shallow water equations based on a Lagrange-Projection type finite volume scheme. This scheme can be interpreted as a two-step algorithm consisting in first solving the shallow water system in Lagrangian coordinates, which is known as the Lagrangian step, and then projecting the results in Eulerian coordinates, which is known as the Projection step. By following this strategy, the acoustic and the transport phenomena can be decoupled and this allows us to design implicit-explicit and large time step schemes in which the CFL restriction is based on the slower transport waves and not on the acoustic ones. In this work we follow the strategy described in [1, 3] to define the LP scheme and [2] to ensure its well-balanced character.

For the Lagrangian step we propose first and second order explicit versions and also, first and second order implicit ones. In the implicit version, we treat the source term implicitly, resulting in a nonlinear system that has to be solved. The Projection step will always be done explicitly.

References:
Arbitrary high order well-balanced WENO finite volume scheme using Flux Globalization

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Abstract:
We are interested in the well-balanced discretization of hyperbolic PDEs with source terms. In the context of preserving stationary states, e.g. lake at rest and moving equilibria, a new formulation of the shallow water system, called Flux Globalization [1] has been introduced. This approach consists in a reformulation of the source term as integral in the flux function and reconstructing the new global flux rather than the conservative variables. The resulting scheme is able to preserve a large family of smooth and discontinuous steady state moving equilibria. In this work, we focus on an arbitrary high order WENO Finite Volume (FV) generalization of the global flux approach of [1]. The most delicate aspect of the algorithm is the appropriate definition of the source flux (integral of the source term) and the quadrature strategy used to match it with the WENO reconstruction of the hyperbolic flux. When this construction is correctly done, one can show that the resulting WENO FV scheme admits exact discrete steady states characterized by constant global fluxes. We also show that, by an appropriate quadrature strategy for the source, we can embed exactly some particular steady states, e.g. the lake at rest for the shallow water equations. It can be shown that an exact approximation of global fluxes leads to a scheme with better convergence properties and improved solutions. The novel method has been tested and validated on classical cases: subcritical, supercritical and transcritical flows.

References:
Implicit and semi-implicit high-order well-balanced methods for one-dimensional systems of balance laws

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Abstract: The goal of this work is to design 1D implicit and semi-implicit high-order well-balanced numerical methods for systems of balance laws. The strategy introduced by two of the authors in [1] for explicit methods is applied, based on the application of a well-balanced reconstruction operator. The well-balanced property is preserved when quadrature formulas are used to approximate the averages and integral of the source term in the cells. This technique is combined with a time discretization method for the time evolution of type RK-IMEX or RK-implicit (see [2]). The methodology will be applied to several systems of balance laws.

References:


In-cell Discontinuous Reconstruction path-conservative methods for non conservative hyperbolic systems - MOOD extension

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Abstract: In the case of systems of conservation laws, in order to ensure the convergence to the right weak solutions of the approximations provided by a method, besides consistency and stability, entropy has to be well controlled. However, in the case of non-conservative systems this is not enough: numerical dissipation effects have to be well controlled.

The theoretical framework of path-conservative methods introduced in [1] facilitates the design of schemes that are formally consistent with the definition of weak solution based on the well-known theory of Dal Maso, LeFloch and Murat. Recently, in [2], an in-cell discontinuous reconstruction technique has been added to first-order path-conservative methods that allows one to capture correctly weak solutions with isolated shock waves. In [3] this technique was extended to second-order accuracy and it was added the well-balanced property.

The main objective of this work is to extend the in-cell discontinuous reconstruction methods to be able to capture well more than one shock waves. This extension is based on the combination of the MOOD strategy and the use of smoothness indicators. Several numerical tests are proposed to validate the methods.

References:


High-order well-balanced finite volume schemes for 1d and 2d shallow water equations with Coriolis forces

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• Jose Antonio García-Rodríguez (jose.garcia.rodriguez@udc.es) Universidad de A Coruña

Abstract: In this work we present two novel well-balanced high-order finite volume schemes for the shallow-water system with Coriolis source term.

On the one hand, we consider the one dimensional system of shallow-water equations. Here, the main objective is to define a high-order ($\geq 3$) exactly well-balanced finite volume scheme for the stationary solutions of the hyperbolic system of equations. By exactly well-balanced we mean a method that is able to preserve the cell-averages of the exact stationary solutions of the system. We follow the general procedure described in [3]. In particular, the method proposed here extends the one presented in [4], where only first and second order schemes are proposed.

On the other hand, we consider the two dimensional system of shallow-water equations. In the literature, we can find different strategies to achieve this goal, see for instance [1, 2]. In our approach, we consider a discrete version of the PDEs that must be satisfied by the stationary solution. The discrete version allows the scheme to be exactly well-balanced for linear velocity fields, while it will be well-balanced for general velocity fields. By well-balanced we mean that it is preserved a high-order approximation of the stationary solution.

References:


Authors:
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Abstract: This work aims to solve a fractional flow model describing oil-water two-phase flow through porous media (see [1]). The mathematical model is formed by an elliptic equation, representing the spatial distribution of the pressure in the reservoir, and a hyperbolic equation describing the space-time evolution of water saturation. A finite element scheme is used to solve the elliptic part while the hyperbolic equation is solved using two different finite volume numerical approaches, namely a Monotonic Upstream-centered Scheme for Conservation Laws, MUSCL-Hancock scheme, and the other one based on a Weighted Essentially Non-Oscillatory (WENO) approach with a First ORder CEntred (FORCE)-α numerical scheme for intercell flux reconstruction. A relevant feature of this work is the study of the effect of the parameter α on the numerical solution of the models considered. We also show that, in the FORCE-α method, when the parameter α increases, the errors diminish and the order of accuracy is more properly attained, as verified using a manufactured solution technique.

References:

Analysis And Numerical Simulation Of 2D Shallow Water Moment Equations

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Abstract: The Shallow Water Moment Equations, introduced in Kowalski and Torrilhon (Commun. Comput. Phys. 25, 3 (2019), 669-702), are an extension of the Shallow Water Equations. They allow for vertical changes in the horizontal velocity, resulting in a system that is more accurate in situations where the horizontal velocity varies considerably over the height of the fluid. Unfortunately, these models lack global hyperbolicity. This has previously been observed for the 1D Shallow Water Moment Equations in Koellermeier and Rominger (Commun. Comput. Phys. 28, 3 (2020), 1038-1084) and we show that the loss of hyperbolicity also occurs in the 2D systems. We derive 2D Hyperbolic Shallow Water Moment Equations by modifying the system matrix. Numerical simulations of dam break problems and smooth test cases using the Hyperbolic Shallow Water Moment Equations yield accurate results, close to the results using non-hyperbolic equations, while guaranteeing hyperbolicity. Time permitting, we present a formulation in cylindrical coordinates and show a simplified model for axisymmetric flow.

References:
Modelling and dispersion relations of layer-averaged non-hydrostatic Euler equations

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Abstract: In this work we present the deduction of several layer-averaged models approximating the Euler equations. The main difference between them is the assumption on the vertical profile of the unknowns inside each layer. The layers are part of the domain, defined between two non-material interfaces, except for the bottom and the free surface. At inner interface mass and momentum transference are considered. We focus on particular weak solutions where the horizontal velocity is layerwise constant or linear on z, and possibly discontinuous across layer interfaces. All the introduced models verify a dissipative energy balance. Finally, an analysis and comparison of dispersive properties of each model will be presented.
An efficient IMEX-DG solver for the compressible Navier-Stokes equations for non-ideal gases

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Abstract:
We propose an efficient, accurate and robust IMEX solver for the compressible Navier-Stokes equation describing non ideal gases with general equations of state. The method, which is based on an $h$–adaptive Discontinuous Galerkin spatial discretization and on an Additive Runge Kutta IMEX method for time discretization, is tailored for low Mach number applications and allows to simulate low Mach regimes at a significantly reduced computational cost, while maintaining full second order accuracy also for higher Mach number regimes. The method has been implemented in the framework of the deal.II numerical library, whose adaptive mesh refinement capabilities are employed to enhance efficiency. Refinement indicators appropriate for real gas phenomena have been introduced. A number of numerical experiments on classical benchmarks for compressible flows and their extension to real gases demonstrate the properties of the proposed method.
Global entropy stability for a class of unlimited second-order schemes for hyperbolic systems of conservation laws

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• Mehdi Badsi (mehdi.badsi@univ-nantes.fr) LMJL - Nantes Université

Abstract: We consider a hyperbolic system of $d \geq 1$ conservation laws

$$\partial_t w + \partial_x f(w) = 0, \quad x \in \mathbb{R}, \quad t > 0,$$

where the unknown $w$ is assumed to belong to $\Omega \subset \mathbb{R}^d$ and $f \in C^1(\Omega, \mathbb{R}^d)$ is a smooth given function whose its jacobian is diagonalizable in $\mathbb{R}$. In order to ensure the uniqueness of the weak solution [3], the system (1) has to be endowed with entropy inequalities which are integrated in space and leads to a global entropy decreasing principle

$$\frac{d}{dt} \int_{\mathbb{R}} \eta(w(x,t)) \, dx \leq 0,$$

where $\eta \in C^2(\Omega, \mathbb{R})$ is a given convex entropy function. From a numerical point of view, the design of an accurate numerical scheme satisfying the inequality (2) at a discrete level is very challenging and often requires additional limitations techniques [1].

In this context, we introduce a new class of second order free of limitation finite volume schemes designed from a perturbation of the standard HLL solver [2]. The perturbation is selected to ensure the accuracy and is discretized in order to prove the global discrete entropy stability property. The accuracy and stability performances of these schemes will be illustrated in the cases of Burger and Euler equations.

References:


A staggered semi-implicit hybrid finite volume / finite element scheme for the shallow water equations at all Froude numbers

Authors:

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Abstract:

We present a novel staggered semi-implicit hybrid finite volume / finite element method for the numerical solution of the shallow water equations at all Froude numbers on unstructured meshes. A semi-discretization in time of the conservative Saint-Venant equations with bottom friction terms leads to its decomposition into a first order hyperbolic subsystem containing the nonlinear convective term and a second order wave equation for the pressure. For the spatial discretization of the free surface elevation an unstructured mesh composed of triangular simplex elements is considered, whereas a dual grid of the edge-type is employed for the computation of the depth-averaged momentum vector. The first stage of the proposed algorithm consists in the solution of the nonlinear convective subsystem using an explicit Godunov-type finite volume method on the staggered dual grid. Next, a classical continuous finite element scheme provides the free surface elevation at the vertices of the primal simplex mesh. The semi-implicit strategy followed in this paper circumvents the contribution of the surface wave celerity to the CFL-type time step restriction, hence making the proposed algorithm well-suited for the solution of low Froude number flows. It can be shown that in the low Froude number limit the proposed algorithm reduces to a semi-implicit hybrid FV/FE projection method for the incompressible Navier-Stokes equations. At the same time, the conservative formulation of the governing equations also allows the discretization of high Froude number flows with shock waves. As such, the new hybrid FV/FE scheme can be considered an all-Froude number solver, able to deal simultaneously with both, subcritical as well as supercritical flows. Besides, the algorithm is well balanced by construction. We show numerical results for a large set of different test cases, comparing with exact and experimental solutions.

References:

Well-Balanced High-Order Discontinuous Galerkin Methods for Systems of Balance Laws

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Abstract: This work introduces a general strategy to develop well-balanced high-order Discontinuous Galerkin (DG) numerical schemes for systems of balance laws. The essence of our approach is a local projection step that guarantees the exactly well-balanced character of the resulting numerical method for smooth stationary solutions. The strategy can be adapted to some well-known different time marching DG discretisations. Particularly, in this article, Runge–Kutta DG and ADER DG methods are studied. Additionally, a limiting procedure based on a modified WENO approach is described to deal with the spurious oscillations generated in the presence of non-smooth solutions, keeping the well-balanced properties of the scheme intact. The resulting numerical method is then exactly well-balanced and high-order in space and time for smooth solutions. Finally, some numerical results are depicted using different systems of balance laws to show the performance of the introduced numerical strategy.
A family of hybrid finite volume / finite element schemes for computational fluid dynamics

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- Michael Dumbser (michael.dumbser@unitn.it) Laboratory of Applied Mathematics, DICAM, University of Trento

Abstract: In this talk, I present the family of semi-implicit hybrid finite volume/finite element (FV/FE) method for solving flows modeled by the incompressible and compressible Navier-Stokes equations, including flows with arbitrary Mach number [1], non-Newtonian and turbulent flows [3]. In order to perform more realistic simulations with complex geometries and finer meshes involving millions of elements, I will show massive parallelization procedures by using the message passing interface (MPI) standard to improve the computational efficiency of the scheme and make the code suitable for use it in high computational capacity infrastructures [2]. The proposed method is applied to some classical benchmark problems for flows from the incompressible limit to all Mach number flows, including non-Newtonian and turbulent flows, comparing the obtained numerical results with available exact or numerical reference solutions, showing an excellent agreement.

References:

Modelling of dry granular flows with weakly non-hydrostatic pressure

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Abstract: Granular flows have been intensely studied in recent years due to their role in many natural hazards. However, its understanding is still a challenge from the physical point of view, and also its mathematical modelling and numerical approximation. In particular, we deal with a complex definition of the stress tensor representing solid-fluid transitions during the flow. Focusing on dry granular flows, several depth-averaged models have been proposed including the $\mu(I)$-rheology ([4]), which is the most accepted rheological law describing the dynamics of these flows (see e.g. [3],[1]).

In this talk we present a simple and efficient depth-averaged model ([2]) for aerial avalanches with a weakly non-hydrostatic pressure. It considers the vertical acceleration but not the contributions coming from the stress tensor. We propose a simple numerical scheme based on a three-step splitting procedure to solve the friction and non-hydrostatic contributions. This scheme is well-balanced for steady solutions at rest with non-constant free surface. Finally, some numerical tests will be presented, in particular comparing with laboratory experiments. We show than the non-hydrostatic model is able to recover the parabolic shape of the front velocity observed in the experiments, contrary to hydrostatic models.

References:


A general multilayer-moment based approach for the vertical approximation of free-surface Euler equations

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Abstract: We present a general technique ([3]) for the vertical discretizations of free-surface Euler equations that generalizes the usual moment and multilayer based approach ([1],[2],[4],[5]). The vertical domain is split into several layers, and inside of them, the velocity is assumed to be piecewise polynomial of arbitrary degree $N$.

Special attention is paid to the multilayer-moment approach with layerwise linear horizontal velocity as a particularly interesting case.

Several numerical tests are presented, devoted to comparing multilayer and moment methods, pointing out some advantages/disadvantages of each approach in terms of efficiency. We also show that the proposed approach allows representing complex vertical structures of the solutions of the hydrostatic free-surface Euler equations.

References:


A first-order hyperbolic reformulation of the Navier-Stokes-Korteweg system

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• Michael Dumbser (michael.dumbser@unitn.it) University of Trento

Abstract: We present a novel first-order hyperbolic reformulation of the Navier-Stokes-Korteweg system, based on the Godunov-Peshkov-Romenski model of continuum mechanics [1], combined with an augmented Lagrangian approach[2], allowing to cast the nonlinear dispersive Euler-Korteweg equations into a first-order hyperbolic system. The latter method is based on a classical penalty method used to approximate the gradient terms in the Lagrangian by a new set of independent variables, for which suitable closure equations are sought. The governing equations for the new introduced degrees of freedom admit curl-free constraints which must be taken into account in order to obtain stable numerical solutions. Thus, we employ here a thermodynamically compatible generalized Lagrangian multiplier (GLM) approach [3], similar to the GLM divergence cleaning introduced by Munz et al. [4] for the Maxwell and MHD equations.

The system of equations is solved at the aid of a high-order ADER Discontinuous Galerkin finite-element scheme with a posteriori subcell finite volume limiting in order to deal with shock waves, discontinuities and steep gradients in the numerical solution. We show numerical results for several standard benchmark problems, including Ostwald ripening in one and two space dimensions, diffuse and dispersive traveling wave solutions.

References:
MS03. PDE models in Biology
Numerical schemes for attraction-repulsion chemotaxis systems

Authors:
- Silvia Frassu (silvia.frassu@unica.it) U. Cagliari
- José Rafael Rodríguez-Galván (rafael.rodriguez@uca.es) Universidad de Cádiz
- Giuseppe Viglialoro (giuseppe.viglialoro@unica.it) U. Cagliari

Abstract:
Since the advent in the 70’s of the biological Keller-Segel models [1] idealizing chemotaxis phenomena, many related variants aroused interest in the mathematical community. Specifically, attraction-repulsion chemotaxis models are of particular relevance and, in addition to the complexity of their mathematical analysis, they present several and interesting difficulties for their numerical approximation.

This work is devoted to exploring some numerical schemes for the consumption-repulsion Keller-Segel equations, based on low order finite elements and flux-corrected transport methods [2]. The behavior of these schemes is investigated from the point of view of preserving positivity of the solutions and also the quality of their approximation to the exact solution. Furthermore, we show some numerical tests where we endorse some recent theoretical results [3] related to boundedness of classical solution of consumption Keller-Segel equations. Finally we explore computationally some concrete cases for which the available theoretical results are not sufficient to shed light.

This work has been supported by the following grants: PGC2018-098308-B-I00 (MCI/AEI/FEDER, UE), US-1381261 (US/JUNTA/FEDER, UE) and P20-01120 (PAIDI/JUNTA/FEDER, UE).

References:
An Upwind DG Scheme Preserving the Maximum Principle for the Convective Cahn-Hilliard Model

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- Francisco Guillén-González (guillen@us.es) Universidad de Sevilla
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Abstract:
The Cahn-Hilliard model has been recently considered for several different applications such as image processing, multicomponent fluid flows, tumor growth processes, etc. However, many of these processes involve transport terms which may produce several numerical difficulties when treated using classical techniques like the Finite Element Method.

This work is devoted to developing a new upwind Discontinuous Galerkin scheme for the convective Cahn-Hilliard model with degenerate mobility which preserves the maximum principle of the continuous model and prevents non-physical spurious oscillations. In particular, we study the analytical properties of the aforementioned scheme.

Furthermore, we present a computational comparison between this scheme and previous schemes found in the literature through different numerical experiments in which the remarkable improvements made by the new scheme are shown. A numerical comparison between the order of convergence of the different spatial schemes is also carried out.

References:
A non-local free boundary problem arising in a model of cell polarization

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• Barbara Niethammer (niethammer@iam.uni-bonn.de) Universität Bonn
• Matthias Röger (matthias.roeger@tu-dortmund.de) TU Dortmund University
• Juan J. L. Velázquez (velazquez@iam.uni-bonn.de) Universität Bonn

Abstract: In this talk, I will present several results for a parabolic non-local free boundary problem that has been derived as a limit of a bulk-surface reaction diffusion system of equations which models cell polarization. In previous work [cf. [1]] we have justified the well-posedness of this problem and we have further proved uniqueness of solutions and global stability of steady states. Yet, these results were not sufficient in order to obtain much insight about the evolution of the support of the solution. Hence, in our recent work we investigate qualitative properties of the free boundary [cf. [2]]. In particular, we conclude that there are necessary and sufficient conditions for the initial data that yield continuity of the support. We show that whenever these assumptions fail, jumps of the set \{u(\cdot, t) > 0\} take place. In addition we provide a complete characterization of the jumps for a large class of initial data.

References:


Nonlocal elliptic system arising from the growth of cancer stem cells

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• Antonio Suárez (suarez@us.es) Dpto. EDAN and IMUS, University of Sevilla, Spain

Abstract: Cancer has been characterized as a collection (complex) of diseases described by uncontrolled growth of cells and development of a tumor that invades the tissue of origin and distant organs.

Furthermore, it is increasingly argued that only a small subset of cancer cells is intrinsically able to initiate and repopulate the tumor. These so-called cancer stem cells (CSCs) have several properties (longevity or even immortality, self-renewal, unlimited proliferation, etc.) and the ability to produce more such cancer stem cells as well as non-stem cancer cells (or tumor cells, TCs).

In [2] a mathematical model of a heterogeneous population of CSCs and TCs is proposed to investigate the “tumor growth paradox”: an increasing rate of spontaneous cell death in TC shortens the waiting time for CSC proliferation and migration, and thus facilitates tumor progression.

In this talk, we study theoretically this model, which includes non-local terms. Mainly, we show the existence of coexistence states. For this, we use the bifurcation method and the theory of the fixed point index in cones. Moreover, in some cases we study the behaviour of the coexistence region, depending on the parameters of the problem.

This work has been supported by the following grants: PGC2018-098308-B-I00 (MCI/AEI/FEDER, UE), US-1381261 (US/JUNTA/FEDER, UE) and P20_01120 (PAIDI/JUNTA/FEDER, UE).

References:


Diffusive Lotka-Volterra models including nonlocal intraspecific and interspecific interactions

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Abstract:
We consider two species inhabiting in $\Omega$, a regular and bounded domain in $\mathbb{R}^N$, whose population densities are denoted by $u(x)$ and $v(x)$. Specifically, we deal with the following system

$$
\begin{align*}
-\Delta u &= u \left( \lambda - u - \int_{\Omega} a(x)u(x)dx + b_1 v + b_2 \int_{\Omega} b(x)v(x)dx \right) \quad \text{in } \Omega, \\
-\Delta v &= v \left( \mu - v - \int_{\Omega} d(x)v(x)dx + c_1 u + c_2 \int_{\Omega} c(x)u(x)dx \right) \quad \text{in } \Omega, \\
u = v &= 0 \quad \text{on } \partial \Omega,
\end{align*}
$$

where $\lambda, \mu \in \mathbb{R}$ denote the growth rates of the species $u$ and $v$, respectively, and $b_1$ and $c_1$ are the local interaction coefficients. When $b_1$ and $c_1$ are negative or positive implies a competition or symbiosis interaction, respectively, and one of them positive and the other negative represents a prey-predator interaction.

We have included two types of non-local terms. The first one appears when one species is not present, that is, in the absence of one species, the other follows a non-local logistic equation. The second non-local term is related with the interspecific non-local interactions.

The main goal is to give existence and non-existence of positive solution results. Specifically, we are interested in determining coexistence regions, that is, subsets $D \subset \mathbb{R}^2$ such that if $(\lambda, \mu) \in D$ then the system possesses at least a coexistence state. A coexistence state is a solution $(u, v)$ of the system such that $u(x), v(x) > 0$ for all $x \in \Omega$. On the contrary, we also provide non-existence regions $N \subset \mathbb{R}^2$, such that, if $(\lambda, \mu) \in N$ then the system does not admit positive solutions. Surely, these regions depend on the type of interactions between the species.

To obtain these results, the nonlocal terms cause difficulties, mainly because classical comparison arguments do not work in general. To overcome this difficulty we have used the bifurcation arguments.

This work has been supported by the grant PGC2018-098308-B-I00 (MCI/AEI/FEDER, UE).
Structural Dynamics and Indirect Effects in the Lotka-Volterra Model

Authors:
- José David Gutiérrez de Alba (josgutdeal@gmail.com) EDAN, University of Sevilla, Spain

Abstract: In population dynamics, structural stability measures the ability of an ecosystem to maintain biodiversity under perturbations of the intrinsic growth rate of species [3, 4]. This type of stability strongly depends on the coefficients of the community matrix. However, it is known that the coefficients that only consider physical interactions between species do not provide an adequate description of the relationships between them [1, 2]. In this presentation we propose a new Lotka-Volterra model that includes indirect interactions between species. After analyzing the existence of feasible stationary points, we conducted a series of simulations to compare the structural stability of both models. The results show that our model with indirect effects has greater structural stability than the original one that only includes direct interactions, suggesting that cooperation and competition might play a different role in structural stability than expected.

This work has been supported by the grant US1381261 (US/JUNTA/FEDER, UE).

References:

On a comparison method for a parabolic-elliptic system of chemotaxis with density-suppressed motility and logistic growth

Authors:
- J. Ignacio Tello (jtello@mat.uned.es) U.N.E.D.

Abstract:
We consider a parabolic-elliptic system of partial differential equations with chemotaxis and logistic growth given by the system

\[
\begin{align*}
  &u_t - \Delta(u\gamma(v)) = \mu u(1 - u), \\
  &\text{Space} - \Delta v + v = u,
\end{align*}
\]

under Neumann boundary conditions and appropriate initial data in a bounded and regular domain \( \Omega \) of \( \mathbb{R}^N \) (for \( N \geq 1 \)), where \( \gamma \in C^3([0, \infty)) \) and satisfies

\[
\gamma(s) > 0, \quad \gamma'(s) \leq 0, \quad \gamma''(s) \geq 0, \quad \gamma'''(s) \leq 0 \quad \text{for any } s \geq 0
\]

\[-2\gamma'(s) + \gamma''(s)s \leq \mu_0 < \mu
\]

\[
\frac{[\gamma'(s)]^2}{\gamma(s)} \leq c, \quad \text{for any } s \in [0, \infty).
\]

We obtain the global existence and uniqueness of bounded in time solutions and the following asymptotic behavior

\[
\|u - 1\|_{L^\infty(\Omega)} + \|v - 1\|_{L^\infty(\Omega)} \to 0, \quad \text{when } t \to +\infty.
\]
Continuous and discrete periodic asymptotic behavior of solutions to a competitive chemotaxis PDEs system

Authors:
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Abstract:
We study the continuous and full discrete versions of a parabolic-parabolic-elliptic system with periodic terms that serves as a model for some chemotaxis phenomena. This model appears naturally in the interaction of two biological species and a chemical. The presence of the periodic terms has a strong impact on the behavior of the solutions. Some conditions on the system’s data are given that guarantee the global existence of solutions that converge to periodical solutions of an associated ODE’s system. Further, we analyze the discretized version of the model using a Generalized Finite Difference Method (GFDM) and we confirm that the properties of the continuous model are also preserved for the resulting discrete model. To this end, we prove the conditional convergence of the numerical model and study some practical examples.

References:
Abstract: The aim of this talk is to analyze an optimal control problem associated to an attractive chemotaxis problem modeling the interaction of a chemical substance with a living organism in a 2D-domain. The systems is of Keller-Segel type with a logistic source term in the equation for the cell density $u$ and a linear production term in the equation for the chemical substance $v$ (see the review [1] for an état de lieu of different chemotaxis models). The control is bilinear and acts over the chemical equation.

First, we prove the existence of a global optimal solution in a weak sense for the control problem. Second, first-order optimality conditions for local optimal solutions are derived by using a Lagrange multiplier theorem.

The main novelty is that the control is only in $f \in L^2$ and then the state cellular density $u$ and the chemical concentration $v$ remains only in the weak setting, without any extra regularity result, which differs from the regularity used in the literature to solve similar optimal control problems (see e.g. [2, 3] for 2D and 3D bilinear control problems associated to repulsive chemotaxis systems).

This work has been supported by the following grants: PGC2018-098308-B-I00 (MCI/AEI/FEDER, UE), US-1381261 (US/JUNTA/FEDER, UE) and P20_01120 (PAIDI/JUNTA/FEDER, UE).

References:


Dual Discrete Finite volume scheme for degenerate parabolic equations

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- El Houssaine Quenjel (el-houssaine.quenjel@centralesupelec.fr) CentraleSupelec
- Ben Mansour DIA (ben.dia@kfupm.edu.sa) CIPR, CPG, King Fahd University of Petroleum & Minerals

Abstract: Nonlinear degenerate parabolic equations are the main core to study some complex problems arising from biology, petroleum engineering and hydrology. In our study, the problem describes the tumour density of the Keller-Segel system or the infiltration of a single fluid through a porous medium with no gravity effects.

We carry out the convergence analysis of a positive DDFV (Dual Discrete Finite Volume) method for approximating solutions of degenerate parabolic equations. The basic idea rests upon different approximations of the fluxes on the same interface of the control volume. Precisely, the approximated flux is split into two terms corresponding to the primal and dual normal components. Then the first term is discretized using a centred scheme whereas the second one is approximated in a non-evident way by an upstream scheme. The novelty of our approach is twofold: on the one hand, we prove that the resulting scheme preserves the positivity and on the other hand we establish energy estimates. Some numerical tests are presented and they show that the scheme in question turns out to be robust and efficient with an accuracy of second order on quadrilateral grids.

References:


Finite element technique for chemotaxis phenomena based on shock capturing

Authors:
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• Jesús Bonilla (jbonilla@lanl.gov) Los Alamos National Laboratory
• Juan Vicente Gutiérrez-Santacreu (juanvi@us.es) Universidad de Sevilla

Abstract: Chemotaxis is the characteristic movement or orientation of an organism or cell along a chemical concentration gradient either toward or away from the chemical stimulus. The first attraction model was derived by Keller–Segel [3] in the early 70’s. Let $\Omega \subset \mathbb{R}^d$, $d = 2$ or $3$, be a bounded domain and let $n$ be its outward-directed unit normal vector to $\partial \Omega$. Find $u : \bar{\Omega} \times [0, \infty) \to (0, \infty)$, the organism density, and $v : \bar{\Omega} \times [0, \infty) \to [0, \infty)$, the chemoattractant density, satisfying

$$\begin{align*}
\partial_t u - \Delta u + \nabla \cdot (u \nabla v) &= 0 \quad \text{in } \Omega \times [0, \infty), \\
\partial_t v - \Delta v + v - u &= 0 \quad \text{in } \Omega \times [0, \infty), \\
\nabla u \cdot n &= 0, \\
\nabla v \cdot n &= 0 \quad \text{on } \partial \Omega \times (0, \infty), \\
[u(0) = u_0, v(0) = v_0] &\quad \text{in } \Omega.
\end{align*}$$

(1)

Solutions to (1) are positivity and mass-preserving, and satisfies an energy law. Furthermore, such solutions, under the condition $\int_\Omega u_0(x) dx \in (0, 4\pi)$, remain uniformly bounded; otherwise, they may blow up either in finite or infinite time.

This talk is about presenting a stabilized finite element method [2] based on a shock capturing technique [1] for which the above desirable properties hold for its discrete solutions. Some numerical results are further shown with regards to non-blowup and blowup phenomena.

This work has been supported by the following grants PGC2018-098308-B-I00 t(MCI/AEI/FEDER, UE) and P20_01120 (PAIDI/JUNTA/FEDER, UE).

References:


On the numerical study of a chemotaxis system modeling tumoral invasion

Authors:
• Viviana Niño-Celis (vivianacelis0519@gmail.com) Universidad Industrial de Santander
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• Diego A. Rueda-Gómez (diaruego@uis.edu.co) Universidad Industrial de Santander
• Élder J. Villamizar-Roa (jvillami@uis.edu.co) Universidad Industrial de Santander

Abstract:
Tissue invasion represents one of the most critical steps in cancer metastasis, which is characterized essentially by four hallmark features, namely, the cancer cell adhesion to the surrounding tissue or extracellular matrix, the secretion of the matrix degrading enzymes and the degradation of extracellular matrix, the migration of the cancer cells, and the proliferation of tumor cells. In order to describe the cancer invasion mechanism, a variety of mathematical models have been proposed in recent years.

In this talk, we focus on the numerical analysis of a PDE-ODE system describing the invasion of host tissue by tumor cells through chemotaxis and haptotaxis phenomena. We propose fully discrete finite element schemes for which we study some properties such that well-posedness, positivity, uniform estimates, error estimates and convergence towards regular solutions. This talk is based on the results obtained in [1, 2].

This work has been partially supported by the grant US-1381261 (US/JUNTA/FEDER, UE).

References:
Theoretical and numerical analysis for a model of glioma invasion

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• Diego A. Rueda-Gómez (diaruego@uis.edu.co) Universidad Industrial de Santander
• Élder J. Villamizar-Roa (jvillami@uis.edu.co) Universidad Industrial de Santander

Abstract: Glioma corresponds to a broad class of brain tumors that arise from glial cells, which are highly invasive and give rise to irregular tumor margins that cannot be identified with sufficient precision by medical imaging. Mathematical modeling of the migration and invasion of tumor cells within tissue, and in particular the modeling of growth and invasion, as well as the evolution and treatment of GBM, has increased significantly in recent years, developing different approaches at discrete, hybrid and continuous levels. In this talk, we consider a general model given by a parabolic system of PDE which describes the proliferation-invasion structure of hypoxic glial cells in the brain and describe some theoretical and numerical results. This talk is based principally on the results obtained in [2].

References:


On a chemotaxis-reaction PDE-ODE system modeling glioblastoma

Authors:
- Antonio Fernández-Romero (afernandez61@us.es) Universidad de Sevilla
- Francisco Guillén-González (guillen@us.es) Universidad de Sevilla
- Antonio Suárez (suarez@us.es) Universidad de Sevilla

Abstract: Glioblastoma (GBM) is a highly proliferative brain tumor. According to quantitative studies, using a large number of magnetic resonances of GBMs ([1,2]), different behaviors appear according to two criteria: 1) The width of the ring forming proliferating cells around necrotic cells [1]. 2) Regular or irregular growth of the GBM surface, [2]. Moreover, from an experimental point of view, there is a correlation between these different behaviors and the average life expectancy [1,2].

In this talk we are going to describe a general model of interaction between three variables (proliferating and necrotic cells, and the vasculature) by means of a hybrid differential system (one partial differential equation for proliferating cells and two ordinary differential equations for necrotic cells and for vasculature). Then, we see that this model is able to capture the different behaviors described above by changing adequately the values of parameters. After describing the model, some theoretical and numerical results will be enunciated for two particular problems (one using a non-linear diffusion [3] and the other one with a chemotaxis term [4]). Finally, some numerical simulations will be show agree with the previous considerations.

This work has been supported by the following grants: PGC2018-098308-B-I00 (MCI/AEI/FEDER, UE), US-1381261 (US/JUNTA/FEDER, UE) and P20_01120 (PAIDI/JUNTA/FEDER, UE).

References:
Analysis of an attraction-repulsion chemotaxis model with saturated signals

Authors:

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- José Rafael Rodríguez Galván (rafael.rodriguez@uca.es) Universidad de Cádiz (Spain)
- Giuseppe Viglialoro (giuseppe.viglialoro@unica.it) Università di Cagliari (Italy)

Abstract: In this talk we focus on an attraction-repulsion chemotaxis model with consumed signals, formulated in bounded and smooth domains $\Omega$ of $\mathbb{R}^n$, with $n \geq 2$. We derive sufficient conditions on its data yielding global and bounded classical solutions to the related zero-flux Cauchy problem.

References:


Criteria towards boundedness for attraction-repulsion Keller–Segel systems

Authors:
• Silvia Frassu (silvia.frassu@unica.it) Università di Cagliari
• Giuseppe Viglialoro (giuseppe.viglialoro@unica.it) Università di Cagliari

Abstract: In this talk we consider two attraction-repulsion chemotaxis systems, one nonlinear and the other linear, and both with produced chemoattractant and saturated chemorepellent. More precisely, our main interest is to find relations between the parameters involving in such problems, which ensure boundedness of solutions.

References:


Numerical schemes for a chemo-attraction and consumption model

Authors:
- Francisco Guillén-González (guillen@us.es) Universidad de Sevilla
- Giordano Tierra (gtierra@unt.edu) University of North Texas

Abstract: During this presentation I will introduce and compare several finite element numerical schemes to approximate a chemo-attraction model with consumption effects, which is a nonlinear parabolic system for two variables; the cell density and the concentration of the chemical signal that the cell feel attracted to.

I will detail the main properties of each scheme, such as conservation of cells, energy-stability and approximated positivity. Moreover, I will present numerical results to illustrate the efficiency of each of the schemes and to compare them with others classical schemes.

This work has been partially supported by the grant US-1381261 (US/JUNTA/FEDER, UE).

References:

MS04. Optimal Control and Inverse Problems
Optimization problems related to antiangiogenic therapy for tumor growth models

Authors:

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• María Peña (mariapenaf@alumnos.unican.es) Facultad de Ciencias, Universidad de Cantabria (SPAIN)

Abstract: Angiogenesis is the natural physiological process of new blood vessel formation and plays a fundamental role in the growth of cancerous tumors. In this work we will study a mathematical model (introduced in [1]) that includes this effect. It consists of a system of two nonlinear ODEs and we will analyze its asymptotic behavior. In addition, the model allows the inclusion of a term for antiangiogenic treatment, whose antitumor action can be optimized using mathematical techniques. Some numerical examples using MATLAB will also be included.

This work is part of María Peña’s undergraduate thesis in Mathematics.

References:

Observability and control of parabolic equations on networks

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Abstract:
During the last decades, the use of networks has been very helpful and effective in the study of pipes, neural systems, the flow of traffic on roads, the global economy or the human circulatory systems. With this talk I would like to show you a contribution to this area from the fields of control theory and inverse problems.

We will consider the propagation of diffusion on a network with loops. Our objective is to control these networks by acting on the system that models the process of the heat diffusion in them, both by open-loop and closed-loop controls, extending in this way the results of [2] and [3] to networks with loops.

The observability of the entire network will be achieved under certain hypotheses about the position of the observation domain. This will be done using a Carleman inequality. Then, we will use that observability to prove the null controllability of the network and to obtain the Lipschitz stability for an inverse problem consisting of retrieving a stationary potential in the heat equation from measurements on the observation domain. The latter will be done following the steps in [3], using the technique of obtaining the inverse problem result from a Carleman inequality.

This talk is based on the work [1].

References:


Multilevel control

Authors:
• Umberto Biccari (umberto.biccari@deusto.es) Fundación Deusto, Bilbao, Spain
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• Carlos Esteve-Yagüe (ce423@cam.ac.uk) Cambridge University, U.K.
• Enrique Zuazua (enrique.zuazua@fau.de) Friedrich-Alexander-Universität, Erlangen, Germany

Abstract:
We discuss the multilevel control problem for dynamical systems, consisting in designing a piece-wise constant control function taking values in a finite-dimensional set. We start by providing a characterization of multilevel controls through an optimal control approach, based on the minimization of a suitable cost functional. We do this by focusing on the Selective Harmonic Modulation problem in power electronic engineering, a practical application of multilevel control, that is what motivates our study [1, 2]. In a second moment, we address the problem of multilevel control for linear ODE systems fulfilling the Kalman rank condition for controllability [3].

References:
On a special class of boundary optimal control problems

Authors:
- Pablo Pedregal (pablo.pedregal@uclm.es) Universidad de Castilla-La Mancha

Abstract:
We introduce a special family of boundary optimal control problems in which we select, in a very precise way, the class of Dirichlet-boundary data that we would allow to compete. Not every possible datum is admitted. We therefore work in a subspace of the form
\[ \mathcal{L} \equiv \mathbb{H} + H^1_0(\Omega) \subset H^1(\Omega) \]
for a proper subspace \( \mathbb{H} \) in \( H^1(\Omega) \). In this form, only boundary data found in \( \mathbb{H} \) are admitted. Other ingredients are typical of a standard optimal control problem like the cost functional or the PDE-state equation. We will explore existence and optimality in this framework, and write a few helpful ideas about the numerical approximation of optimal solutions. The problem is motivated by the new type of boundary conditions examined in [1].

References:
Nonlocal basis pursuit: Nonlocal optimal design of conductive domains in the vanishing material limit

Authors:
• José Carlos Bellido (JoseCarlos.Bellido@uclm.es) Departamento de Matemáticas, Universidad de Castilla-La Mancha
• Anton Evgrafov (anev@math.aau.dk) Department of Mathematical Sciences, Aalborg University

Abstract: We consider the problem of optimal distribution of a limited amount of conductive material in systems governed by local and non-local scalar diffusion laws. Of particular interest for these problems is the study of the limiting case, which appears when the amount of available material is driven to zero. Such a limiting process is of both theoretical and practical interest and continues to be a subject of active study. In the local case, the limiting optimization problem is convex and has a well understood basis pursuit structure. Still this local problem is quite challenging both analytically and numerically because it is posed in the space of vector-valued Radon measures.

With this in mind we focus on identifying the vanishing material limit for the corresponding nonlocal optimal design problem. Similarly to the local case, the resulting nonlocal problem is convex and has the basis pursuit structure in terms of nonlocal anti-symmetric two-point fluxes. In stark contrast with the local case, the nonlocal problem admits solutions in Lebesgue spaces with mixed exponents. We also investigate the limit when the nonlocal interaction horizon is driven to zero, the “vanishing material limit” nonlocal problems provide a certain estimate for the corresponding local measure-valued optimal design problem.

References:


Topology optimization problems with connectivity constraints

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- Ernesto Aranda (ernesto.aranda@uclm.es) UCLM
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- David Ruiz (david.ruiz@uclm.es) UCLM

Abstract: In this talk we present a new method to impose connectivity constraints in topology optimization problems. These constraints allow to design optimal structures without enclosed holes ([2]), or efficient piezoelectric transducers having two opposite polarities, where connectivity in each phase is crucial to avoid isolated features of like-polarity which lead to more difficult wiring schemes ([3]).

The method proposed has been developed in two different contexts: we firstly consider a discrete approximation based on a well known result of graph theory (cf. [4]), and secondly, we use a continuous approximation using a characterization of the connectivity through the eigenvalues of the Neumann Laplacian ([1]).

References:

Topology optimization of flexoelectric materials considering a micromorphic approach

Authors:
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- Jesús Martínez-Frutos (jesus.martinez@upct.es) Universidad Politécnica de Cartagena, Spain
- Antonio J. Gil (a.j.gil@swansea.ac.uk) Swansea University, UK

Abstract:
Flexoelectricity is a two-way coupling mechanism between electric polarisation and inhomogeneous deformations, only relevant at small length scales [1, 2]. Few works exist on the effective application of TO techniques for the design of energy harvesters based on the principle of flexoelectricity [3]. The latter works are restricted to piezoelectric crystals and ceramics, hence, exhibiting extremely small deformations. Dielectric elastomers, devoid from the piezoelectric effect, can also exhibit flexoelectricity in the microscale. However, unlike their crystal/ceramic counterparts, they are capable of undergoing large deformations, and therefore, have been identified as ideal candidates for their use as flexoelectric energy harvesters.

As shown in [3], embedding TO algorithms with the aim of maximising an efficiency-based objective function, typically measuring the ratio between the global (integral) mechanical energy and the global electric energy generated through flexoelectricity, leads to designs characterised by a poor structural performance. This mechanical deficiency is indeed promoted by the inherent flexoelectric nature of the material, which promotes the development of hinges with highly localised large strain gradients. In order to remedy the above shortcoming, we propose a new TO framework for the robust (safe) design of flexoelectric actuators with the following key novelties: (a) consideration of dielectric elastomers, hence, of finite strain scenarios; (b) definition of objective functions considering a weighted combination of efficiency-based measures and p-norm/KS aggregation/regularisation functions of the Von Mises stress distribution within the material.

References:
Load matrix recovery from scattering data in linear elasticity

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Abstract:
We study the numerical approximation of the inverse scattering problem in the two-dimensional homogeneous isotropic linear elasticity with an unknown linear load given by a square matrix.

For both backscattering data and fixed-angle scattering data, we propose a convergent trigonometric collocation method to obtain numerical approximations of the so-called Born approximations introduced in [1]-[2] to recover singularities of the load. Numerical simulations show that the Born approximation not only recovers the singularities of the load but also provides a fairly good approximation of its four components themselves when they are not too large.

As in more classical scalar scattering problems, the Born approximation is constructed from the Fourier transform of the potential. However, in this case the vectorial nature of the problem makes this construction intricate, combining different scattering amplitudes at different energies and in different directions. The construction is also different for backscattering data and fixed-angle scattering data.

Based on this Born approximation we also propose new iterative algorithms that rapidly converge to the unknown load matrix when it is not too large. We illustrate this convergence with numerical simulations.

References:
Exponential turnpike property for fractional parabolic equations with non-zero exterior data

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Abstract: In this talk, we consider averages convergence as the time-horizon goes to infinity of optimal solutions of time-dependent optimal control problems to optimal solutions of the corresponding stationary optimal control problems. Control problems play a key role in engineering, economics and sciences. To be more precise, in climate sciences, often times, relevant problems are formulated in long time scales, so that, the problem of possible asymptotic behaviors when the time-horizon goes to infinity becomes natural. Assuming that the controlled dynamics under consideration are stabilizable towards a stationary solution, the following natural question arises: Do time averages of optimal controls and trajectories converge to the stationary optimal controls and states as the time-horizon goes to infinity? This question is very closely related to the so-called turnpike property that shows that, often times, the optimal trajectory joining two points that are far apart, consists in, departing from the point of origin, rapidly getting close to the steady-state (the turnpike) to stay there most of the time, to quit it only very close to the final destination and time. In the present talk we deal with heat equations with non-zero exterior conditions (Dirichlet and nonlocal Robin) associated with the fractional Laplace operator \((-\Delta)^s\) \((0 < s < 1)\). We will show the turnpike property for the nonlocal Robin optimal control problem and the exponential turnpike property for both Dirichlet and nonlocal Robin optimal control problems.
Acoustic Full-Waveform Inversion via Optimal Control: First- and Second-Order Analysis

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- Irwin Yousept (irwin.yousept@uni-due.de) Universität Duisburg-Essen

Abstract: This talk discusses an optimal control approach for acoustic full-waveform inversion (FWI) to reconstruct the speed wave parameter entering the hyperbolic acoustic PDE model in the coefficient of the second-order time derivative of the acoustic pressure. We develop a novel technique accounting for an auxiliary first-order system. In contrast to the original state equation, the underlying control parameter appears in the auxiliary system not only as the coefficient of the time derivative but also as the initial data under the image of the solution operator for a specific elliptic problem. On this basis, we construct an adjoint state explicitly using the corresponding dual semigroup, leading to necessary optimality conditions with a low adjoint regularity such that no Sobolev smoothing effect occurs in the optimal solution. The final part of the talk is devoted to the second-order analysis of the optimal control approach. Under specific regularity and compatibility assumptions on the given data with domains of dimension two or three, we propose a second-order sufficient condition taking into account the square root of the cubic $L^3$-norm and strongly active sets. The proposed condition guarantees local optimality with $L^3$-cubic growth in an $L^\infty$ neighborhood. A numerical test based on a synthetic configuration with non-smooth data is provided illustrating the performance of the control approach.
Inverse problem for Hamilton-Jacobi equations: projections onto the reachable set and semiconcave envelopes

Authors:

- Carlos Esteve Yagüe (ce423@cam.ac.uk) University of Cambridge

Abstract:

In this talk, we consider the inverse problem of reconstructing the initial condition from the solution to a Hamilton-Jacobi equation, given at some positive time. First of all, we describe the reachable set, i.e. the set of functions for which there exists at least a compatible initial condition. Secondly, we discuss the possibility of having multiple inverse designs for the same given target function. Finally, for a given unreachable target, we compare two different projections onto the reachable set. Namely, we study the orthogonal projection, which minimizes the $L^2$-distance to the target, and the so-called backward-forward projection, which gives the smallest reachable function bounded from below by the target. This is a joint work with Enrique Zuazua.

References:


MS05. Iterative Processes and Non Linear Equations
Familia de métodos iterativos de resolución de sistemas no lineales basada en el método de Homeier

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Abstract: En los últimos años, con el auge de las herramientas computacionales, ha proliferado el diseño de métodos iterativos para resolver ecuaciones y sistemas no lineales. Estos métodos permiten obtener la solución aproximada de numerosos problemas presentes en campos como la Ingeniería, las Ciencias Naturales o la Economía, entre otros.

Homeier [1] diseñó un método de convergencia cúbico para ecuaciones no lineales de dos pasos, sobre el que Cordero et al. [2] introdujeron parámetros y analizaron su comportamiento dinámico.

En este trabajo – de forma similar a otros autores – extendemos la idea de Homeier sobre la resolución de ecuaciones a la resolución de sistemas no lineales. La novedad reside en la introducción de un tercer paso a partir del cual obtenemos una familia de métodos de orden cuatro. Además, un miembro muy concreto de esta familia alcanza el orden de convergencia cinco. Analizamos la estabilidad de la familia de métodos iterativos a través de la dinámica real multidimensional y verificamos la convergencia de diferentes miembros a través de tests numéricos que corroboran los resultados teóricos.

Acknowledgements: This research was supported by Spanish Ministerio de Ciencia, Innovación y Universidades PGC2018-095896-B-C22.

References:


Métodos iterativos para la obtención simultánea de raíces de ecuaciones no lineales

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Abstract: En la mayoríá de los problemas que aparecen en Ciencias e Ingeniería necesitamos resolver ecuaciones no lineales de la forma \( f(x) = 0 \). En estos casos, los métodos iterativos son especialmente útiles. En ellos, dada una estimación inicial de la solución, se genera una sucesión de iterados que, bajo ciertas condiciones, converge a la raíz de la ecuación no lineal.

Normalmente, los métodos iterativos se centran en la obtención de una única solución del problema, pero a veces necesitamos obtener todas las soluciones. Para dar respuesta a esta necesidad, surgen los métodos iterativos de raíces simultáneas, en los que, dado un conjunto de estimaciones iniciales se obtienen sucesiones de iterados que, bajo ciertas condiciones, convergen a todas las raíces de la ecuación de forma simultánea. Podemos encontrar algunos métodos iterativos para raíces simultáneas existentes en la literatura en [1] y [2].

En este trabajo, nos planteamos si existe un procedimiento general que permita diseñar un método de raíces simultáneas a partir de cualquier esquema iterativo. En nuestra propuesta hemos añadido un paso a un método iterativo cualquiera de orden de convergencia \( p \), obteniendo un esquema que permita encontrar raíces simultáneas con velocidad de convergencia \( 2p \).

El alto orden de los métodos resultantes hace que la comparación de dichos esquemas con otros conocidos sea especialmente significativa. En las pruebas realizadas, los resultados numéricos de los métodos propuestos son más eficientes, especialmente sobre funciones no polinómicas, que las de los procedimientos ya existentes.

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References:


Extensión del análisis discreto multidimensional para métodos vectoriales con memoria

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Abstract: Los métodos iterativos para aproximar las soluciones de sistemas de ecuaciones no lineales son una herramienta fundamental para la resolución de problemas en Ciencias e Ingeniería, ya que en la mayoría de los problemas no es posible resolver estos sistemas de forma exacta. Por este motivo, se utilizan los métodos iterativos, ya que, al dar una estimación inicial lo suficientemente cercana a la solución, obtienen una aproximación de la misma.

El análisis de la dependencia de los esquemas iterativos de la estimación inicial se lleva a cabo mediante técnicas de dinámica discreta, real o compleja. Estas técnicas pueden utilizarse para analizar las propiedades cualitativas de nuevos métodos iterativos sin memoria (véase, por ejemplo, [1]) o con memoria (véase, por ejemplo, [2]). También cambian si el método es multidimensional, como podemos ver en [3].

En este trabajo sentamos las bases del análisis dinámico de los métodos iterativos con memoria para la aproximación de las soluciones de sistemas no lineales. Las técnicas existentes, tanto de dinámica discreta compleja como real, no están definidas para superar la alta dimensionalidad de las funciones racionales implicadas. Para comprobar la efectividad del procedimiento propuesto, se analiza el rendimiento de los métodos multidimensionales de Kurchatov y Steffensen en sistemas polinómicos no lineales acoplados y no acoplados. Los resultados obtenidos muestran la aplicabilidad de esta técnica y abren un amplio campo de investigación en el futuro próximo.

Acknowledgements: Esta investigación está subvencionada por el Ministerio de Ciencia, Innovación y Universidades PGC2018-095896-B-C22, por el proyecto ADMIREN de la Universidad Internacional de La Rioja (UNIR) y por la Universitat Politècnica de València Contrato Predoctoral PAID-01-20-17 (UPV).

References:

Chebyshev-type methods for the matrix p-th root

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Abstract: The main goal of this paper is to approximate the principal p-th root of a matrix by using Chebyshev-type iterative methods. We analyze the semi-local convergence and the speed of convergence of these methods. We present stable versions of the algorithms. We test numerically the methods checking the numerical robustness and stability. We present a comparison with the Newton and the Halley methods. These two algorithms are basically the iterative methods proposed in the literature to solve this problem.

References:
Funciones generadoras de Catalan

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Abstract: En este trabajo que presentamos estudiamos la solución de la ecuación cuadrática $TY^2 - Y + I = 0$ donde $T, Y \in B(X)$, donde denotamos por $B(X)$ el conjunto de operadores lineales y acotados en $X$, siendo $X$ un espacio de Banach e $I$ la identidad en $X$. Describimos el espectro y la resolvente de $Y$ en términos de $T$. Veremos que si el operador $4T$ es de potencias acotadas uniformemente, una solución está dada por

$$C(T) := \sum_{n=0}^{\infty} C_n T^n$$

donde denotamos por $(C_n)_{n \geq 0}$ la sucesión de los números de Catalan [3]:

$$C_n = \frac{1}{n+1} \binom{2n}{n}, \quad n \geq 0.$$ 

Estos números aparecen en una amplia variedad de problemas, véase por ejemplo [1, 3, 4]. Nos referiremos a la serie $C(T)$ como la función generadora de Catalan. Mostraremos que la función generadora de Catalan, $C(T)$, admite una representación integral que involucra el operador resolvente $(\lambda - T)^{-1}$, [2]. También, desde el punto de vista numérico aproximaremos una solución para un caso particular de esta ecuación cuadrática con un método iterativo que involucra a los números de Catalan.

References:

Generalization of the secant method. High order methods free of derivatives.

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Abstract:
In this work we present an immediate generalization of the Secant and Mülller methods by using higher order polynomials. We proved the results leading to the determination of the order of convergence of these methods, and we analyze its efficiency. Since the convergence order of these methods remains always lower than 2, we propose two possible algorithms to define alternative methods with increasing orders of convergence and also free of derivatives. The orders of convergence of these new methods conform a non bounded and strictly increasing sequence. All these issues will be also tested with numerical experiments.

References:
Iterative method for multiple roots without knowing multiplicity

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Abstract: Iterative methods are a widely used tool for solving nonlinear equations \( f(x) = 0 \). One of them is Newton’s method, which has the following expression:

\[
x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad \text{for } k = 1, 2, 3\ldots
\]

To ensure the convergence of this method, we require that the derivative of the function \( f \) evaluated at the solution is non-zero, that is, that the solution is a single zero. We then have problems when applying Newton’s method to solve equations as simple as \((x - 1)^2\).

For this reason, methods for multiple roots appear, although we have a problem with many of these methods, and that is that their iterative expression depends on the multiplicity of the root, so we must know this multiplicity.

In the article [1], they replace \( f(x) \) by \( g(x) \) with \( g(x) = \frac{f(x)}{f'(x)} \), thus avoiding the need to know the multiplicity. In our case, we will apply this change of function to Kurchatov’s method to obtain a method for multiple roots without derivatives, and we will also check that the order of Kurchatov’s method still stands at 2.

We will perform several numerical experiments with Kurchatov’s method for multiple roots, comparing the results with other methods of the same characteristics already known

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References:
Some progress in the study of Schröder’s method applied to cubic polynomials.

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Abstract:

1. The Schröder’s method was introduced in [4]. This work was published by Ernst Schröder, where he also exposes two famous families of iterative methods, the so-called Schröder families of the first and second type.

2. The dynamics of this method applied to polynomials with 2 differentiated roots has been studied in detail by Galilea and Gutiérrez in [2].

3. A first approximation of the study of the method applied to cubic polynomials has already been made in [1].

4. The aim of this paper is to advance in the study of Schröder’s method applied to cubic polynomials. We will use tools such as the parameter plane, studied by Gutiérrez, Varona in [3] and we will try to characterise when the basin of attraction of one of the three roots dominates the other two.

References:

New results on the dynamics of the Chebyshev-Halley family of iterative methods

Authors:
• José M. Gutiérrez (jmguti@unirioja.es) Universidad de La Rioja

Abstract: The well known Chebyshev-Halley family of iterative methods for solving scalar equations \( f(z) = 0 \) was firstly introduced by Werner [3] in 1980. Since then, it has been considered by many authors in the field of the numerical solution of nonlinear equations. The idea is to construct a sequence \( z_{n+1} = G_\alpha(z_n), n \geq 0, \) where

\[
G_\alpha(z) = z - \left(1 + \frac{L_f(z)}{2(1 - \alpha L_f(z))}\right) \frac{f(z)}{f'(z)}, \quad L_f(z) = \frac{f(z)f''(z)}{f'(z)^2}.
\]

(1)

In this work we are interested in the study of the complex dynamics of the methods introduced in (??). In particular, we characterize the existence of strange fixed points (that is, fixed points of \( G_\alpha(z) \) that are not roots of \( f(z) \)) in terms of the parameter \( \alpha \). In addition, we also analyze the number of free critical points for the methods in (1). This fact plays a key role in the graphic representation of the parameter plane of each method and, therefore, in their dynamical analysis, as we can see in [1] or [2], for instance.

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References:


Iteration of rational Hopf-endomorphisms for graphical representation of basins of attracting \( n \)-cycles

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Abstract:

In this talk we will present a new collection of algorithms dedicated to compute the basins of attraction of any complex rational map, and to study the discrete-time dynamical behaviour of its attracting \( n \)-cycles. This is a relevant matter when studying some of the most extensively used numerical methods to approximate solutions of non-linear polynomial equations. For example, when one applies Newton’s method to a polynomial, it induces a rational map for which the roots of the polynomial are super-attracting 1-cycles (fixed points).

In order to compute the basins of attraction, we define a function that is constant on each basin, and which depends on the notion of spherical derivative of the given rational map. This way, we can divide the Riemann Sphere (the plane of complex numbers adding the infinity point) into the different basins of attraction and the Julia set.

From a scientific programming point of view, this new collection of algorithms solves some of the computational problems that often arise in Numerical Analysis, like overflows or mathematical indeterminations. We achieve this by considering the Hopf-endomorphism induced by the given rational map, and iterating it over the complex projective line \( P^1(\mathbb{C}) \). This approach also allows us to easily work with the infinity point. Since this kind of computations are often very heavy, we chose Julia Language in order to implement our algorithms, due to its efficiency, speed and proper syntax for mathematics.

The implementation of the presented algorithms can be found in the Lyapunov Cycle Detector module, available in the following GitHub repository: github.com/LCD.

References:


High order method for the SVD

Authors:

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Abstract:

In this talk, we present a class of high order methods to approximate the singular value decomposition of a given complex matrix (SVD). To the best of our knowledge, only methods up to order three appear in the literature. A first part is dedicated to define and analyse this class of method in the regular case, i.e., when the singular values are pairwise distinct. The construction is based on a perturbation analysis of a suitable system of equation associated to the SVD (SVD system). More precisely, for an integer $p$ be given, we define a sequence which converges with an order $p+1$ towards the left-right singular vectors and the singular values if the initial approximation of the SVD system satisfies a condition which depends on three quantities: the norm of initial approximation of the SVD system, the greatest singular value and the greatest inverse of the modulus of the difference between the a singular value. From a numerical computational point of view, this furnishes a very efficient simple test to prove and certify the existence of a SVD in neighborhood of the initial approximation. The second part of this study extends the method in the general case, i.e., when there are clusters of singular values. Moreover numerical experiments confirm the theoretical results both in the regular case of distinct singular value and in the singular case of multiple or clustered singular values.

References:

An unfeasibility view of neural network learning

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Abstract: We define the notion of a continuously differentiable perfect learning algorithm for multilayer neural network architectures and show that such algorithms do not exist provided that the length of the data set exceeds the number of involved parameters and the activation functions are logistic, tanh or sin.

We observe that like in the case of automatic learning by neural networks the astronomical calculations based on epicycles in the Ptolemaic system produce only approximative results. Thus, automatic learning is in fact not a modern concept, it has a long history. In particular, ancient greek astronomy was marked by a deep epistemological discussion about the scope of automatic learning techniques.

References:


MS06. ALAMA: Linear Algebra, Matrix Analysis and Applications
Gaussian quadrature formulae and total positivity

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Abstract:
An algorithm to compute the nodes and weights of Gaussian quadrature formulae for some families of classical orthogonal polynomials is presented. In particular, the families of the Laguerre polynomials and the Chebyshev polynomials on [0,1] are considered.

The algorithm is based on linear algebra techniques. Its first stage is the fast and accurate computation of the bidiagonal decomposition of the Jacobi matrices associated to the orthogonal polynomials. Then, starting from this bidiagonal decomposition, the eigenvalues and eigenvectors of the corresponding Jacobi matrix are accurately computed. The numerical experiments corroborate the accuracy of the developed algorithm, which is a direct consequence of the exploitation of the total positivity of the Jacobi matrices.

References:
Linear algebra over linear control systems

Authors:
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Abstract:
Maxwell’s seminal work [3] introduced mathematical tools to study control systems. Cybernetics introduced the notion of feedback action, or closed-loop, in order to control effectively a system. Linear control systems are of special interest in control theory.

The notion of feedback morphism is recently introduced [2]. A feedback morphism is a linear map between (state-spaces) of linear systems such that dynamics and control are preserved. Invertible feedback morphisms are exactly the classical feedback equivalences.

In this talk we introduce the linear algebra machinery to feedback morphisms. Kernels, cokernels, images, and natural decompositions of linear systems and feedback morphisms are introduced in such a way that the pair (Linear systems, Feedback morphisms) arises as an additive category that is preabelian [1] (there exist kernels and cokernels), and where classical decompositions of linear systems (Kalman, Brunovsky) can be studied from feedback morphisms by using classical linear algebra results on that category.

References:

On bundles of matrix pencils under strict equivalence

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Abstract:
Bundles of matrix pencils (under strict equivalence) are sets of pencils having the same Kronecker canonical form, up to the eigenvalues (namely, they are an infinite union of orbits under strict equivalence). The notion of bundle for matrix pencils was introduced in the 1990’s, following the same notion for matrices under similarity, introduced by Arnold in 1971, and it has been extensively used since then. Despite the amount of literature devoted to describing the topology of bundles of matrix pencils, some relevant questions remain still open in this context. For example, the following two:

(a) provide a characterization for the inclusion relation between the closures (in the standard topology) of bundles; and

(b) are the bundles open in their closure?

In this talk, we present an explicit answer to these two questions. In order to get this answer, we also review and/or formalize some notions and results already existing in the literature. If time permits, we will also show that bundles of matrices under similarity, as well as bundles of matrix polynomials (defined as the set of \( m \times n \) matrix polynomials of the same degree having the same spectral information, up to the eigenvalues) are open in their closure.
High relative accuracy for some subclasses of $P$-matrices

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Abstract: Achieving high relative accuracy (HRA) can be a remarkable property for a numerical algorithm [2]. In this talk, we will present a survey on some of the recent advances in the development of accurate methods for two important subclasses of $P$-matrices: nonsingular $M$-matrices and nonsingular totally positive matrices. We will show how the strong structure of these classes of matrices can be represented by a good parametrization [1, 3, 4]. Then, these parametrizations can be used to achieve great accuracy when solving some of the most common problems in numerical linear algebra. We illustrate this fact with some classes of matrices for which HRA has been assured.

References:

Accurate computations with totally positive Lagrange-Vandermonde matrices

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Abstract:
Lagrange-Vandermonde matrices are a generalization of Vandermonde matrices obtained when considering a Lagrange-type basis for the space of the algebraic polynomials of degree less than or equal to $n$. Since these structured matrices are usually ill-conditioned, standard algorithms to solve numerical linear algebra problems involving them fail to provide accurate results.

In this talk we present accurate and efficient algorithms to solve some numerical linear algebra problems with this class of matrices, such as the computation of eigenvalues and, in the rectangular case, the computation of singular values. An essential stage of the algorithms is the fast and accurate computation of the bidiagonal decomposition of the Lagrange-Vandermonde matrices. Numerical experiments showing the good performance of our approach are also included.

References:


Accurate computations with Bernstein mass matrices

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Abstract:
Certain bases with nice properties for the interactive design of curves and surfaces are not orthogonal with respect to a given inner product. In some cases, to overcome this inconvenience, they are transformed into other orthogonal bases through a transformation matrix, which is a Gram matrix.

In the space of polynomials of degree less than or equal to \(n\) on the interval \([0, 1]\), the \((n + 1) \times (n + 1)\) Gram matrix of the corresponding Bernstein basis is usually called Bernstein mass matrix. This matrix is the change of basis matrix from the dual Bernstein basis to the Bernstein basis.

The inversion of the Bernstein mass matrices is required for the approximation, in the least-squares sense, of curves by linear combinations of control points and Bernstein polynomials. Unfortunately, as the degree of the bases increases, the corresponding Bernstein mass matrices become ill-conditioned. Then, some algebraic problems such as the computation of their eigenvalues, their singular values, their inverse matrix, or the solution of some systems of linear equations do not achieve accurate results. Therefore, computations with high relative accuracy (HRA) using Bernstein mass matrices is an important issue in numerical linear algebra.

In this talk, we are going to consider Bernstein mass matrices, as well as their principal submatrices, corresponding to a general inner product

\[
\langle f, g \rangle := \int_0^1 t^\alpha (1-t)^\beta f(t)g(t) \, dt, \quad \alpha, \beta > -1.
\]

A bidiagonal factorization of them will be provided. We shall illustrate that, by means of the proposed decomposition, the resolution of the above mentioned algebraic problems can be achieved with HRA and consequently, the accuracy of the obtained solutions does not considerably decrease with the dimension of the matrices.
Totally Positive Matrices and Gaussian Markov Random Fields

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Abstract: Totally positive matrices, which are matrices with non-negative minors, have many important applications in several fields such as Statistics, Approximation Theory, Mechanics, Economy or Computer Aided Geometric Design (see [1, 4]). In the context of Probability and Statistics, covariance matrices are widely used in both theoretical and applied problems (see [2]). In particular, the computation of its eigenvalues and eigenvectors, the Cholesky factorization and the calculation of the inverse are problems that naturally appear when dealing with covariance matrices. Since total positivity is a desirable property regarding numerical computations (computations with this type of matrices can be achieved with High Relative Accuracy in some cases, [3]), the study of conditions under which a covariance matrix is totally positive is of interest. In this work, some properties of totally positive covariance matrices are studied and a sufficient condition for total positivity is provided, linking the problem to Gaussian Markov Random Fields and the sparseness of the precision matrix. Some cases for which this condition is also necessary are commented.

References:

Backward stability in rational eigenvalue problems solved via block Kronecker linearizations

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Abstract: Rational eigenvalue problems have received considerable attention in the last decade, since they appear in many applications and they are used to approximate general nonlinear eigenvalue problems. Each rational eigenvalue problem is defined via a rational matrix \( R(\lambda) \), i.e., a matrix whose entries are rational functions in the variable \( \lambda \), which can be represented in different forms. We study the backward stability of running a backward stable eigenstructure solver on a pencil \( S(\lambda) \) that is a strong linearization of an arbitrary rational matrix \( R(\lambda) \) expressed in the form \( R(\lambda) = D(\lambda) + C(\lambda I_ℓ - A)^{-1}B \), where \( D(\lambda) \) is a polynomial matrix and \( C(\lambda I_ℓ - A)^{-1}B \) is a minimal state-space realization. We consider the family of block Kronecker linearizations of \( R(\lambda) \), which have the following structure

\[
S(\lambda) := \begin{bmatrix}
M(\lambda) & \hat{K}_2^T C & K_2^T (\lambda) \\
Space B \hat{K}_1 & A - \lambda I_ℓ & 0 \\
K_1(\lambda) & 0 & 0
\end{bmatrix},
\]

where the blocks have some specific structures. Backward stable eigenstructure solvers applied to \( S(\lambda) \) will compute the exact eigenstructure of a perturbed pencil \( \hat{S}(\lambda) := S(\lambda) + \Delta S(\lambda) \) and the special structure of \( S(\lambda) \) will be lost. In order to link this perturbed pencil with a nearby rational matrix, we construct a strictly equivalent pencil \( \tilde{S}(\lambda) = (I - X)\hat{S}(\lambda)(I - Y) \) that restores the original structure, and hence is a block Kronecker linearization of a perturbed rational matrix \( \tilde{R}(\lambda) = \tilde{D}(\lambda) + \tilde{C}(\lambda I_ℓ - \tilde{A})^{-1}\tilde{B} \), where \( \tilde{D}(\lambda) \) is a polynomial matrix with the same degree as \( D(\lambda) \). Moreover, we bound appropriate norms of \( \tilde{D}(\lambda) - D(\lambda), \tilde{C} - C, \tilde{A} - A \) and \( \tilde{B} - B \) in terms of an appropriate norm of \( \Delta S(\lambda) \). These bounds may be inadmissibly large, but we introduce a scaling that allows us to make them satisfactorily tiny. Thus, for this scaled representation, we prove that the \( QZ \) algorithm computes the exact eigenstructure of a rational matrix \( \tilde{R}(\lambda) \) that can be expressed in exactly the same form as \( R(\lambda) \) with the parameters defining the representation very near to those of \( R(\lambda) \). This shows that this approach is backward stable in a structured sense.
Compensation of positivity in the Symmetric Nonnegative Inverse Eigenvalue Problem

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Abstract:

The NIEP (Nonnegative Inverse Eigenvalue Problem) deals with characterizing the possible spectra of entrywise nonnegative matrices. It is a long-standing open problem in Matrix Analysis which allows for different variations, depending on which kind of nonnegative matrix is chosen to realize the spectrum. The symmetric version, the so-called SNIEP, in which the realizing matrix has to be symmetric, has received quite some attention of late.

Any list of real numbers which is the spectrum of a symmetric nonnegative is said to be realizable. Among all realizable lists a subclass was identified as those ‘realizable by compensation’ (in short, C-realizable), which was shown in [1] to include most of subclasses known so far associated with sufficient realizability conditions.

In this talk we present a combinatorial characterization of C-realizable lists, first for the special case of zero-sum lists [2], and then for arbitrary ones with nonnegative sum. One of the consequences of this characterization is that the set of zero-sum C-realizable lists is a union of polyhedral cones whose faces are described by equations involving only linear combinations with coefficients 1 or −1 of the entries in the list. Lists with positive sum are C-realizable if and only if there exists a shifted version with zero sum satisfying the equations mentioned above.

References:


Tuned preconditioners for linear systems associated with the time-dependent neutron $\text{SP}_N$ equations

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Abstract:
The time-dependent $\text{SP}_N$ equations can be used to obtain accurate approximations of the evolution of the neutronic power inside of a nuclear reactor core. The spatial discretization yields a stiff semi-discrete system of ordinary differential equations. One of the most used strategies to solve these types of systems of ODEs is to apply implicit schemes that implies solving many linear systems with nonsymmetric coefficient matrices. Krylov subspace methods, such as the GMRES method, can be used to solve these systems with fast convergence provided that a good preconditioner is used [1].

Preconditioners based on incomplete factorization of the matrices work well but they need the allocation of the matrices entries and this can lead to problems of lack of CPU memory for realistic nuclear problems. This work studies different strategies to construct preconditioners by means of low-rank updates of an initial preconditioner, which can be applied by matrix-vector products [2], thus allowing a ‘matrix-free’ implementation. In particular, we consider tuned preconditioners based on the ‘bad’ and ‘good’ Broyden’s methods and spectral preconditioners. The efficiency of these updating methodologies is tested for different basis of the subspace considered to construct the update. The numerical results show the effectiveness of the resulting updated preconditioners for different nuclear benchmark transients, even when the initial preconditioner is taken as simply the diagonal of the coefficient matrix.

References:


MS07. NEW TRENDS ON THE 1-LAPLACIAN
The 1-Laplacian in Metric Graphs

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Abstract: Our aim is to study the Total Variation Flow in Metric Graphs. First, we define the functions of bounded variation in Metric Graphs and their total variation, we also give an integration by parts formula. We define the 1-Laplacian in Metric Graphs and we prove existence and uniqueness of solutions of the Cauchy problem for the 1-Laplacian and that the solutions reach the stationary state in finite time. Moreover, we obtain explicit solutions.
Nonlocal nonlinear diffusion problems with nonlinear boundary conditions

Authors:
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Abstract: The study of partial differential equations in nonlocal frameworks and graphs has attracted a great deal of interest in recent years. In this talk we give results on existence and uniqueness of mild and strong solutions of nonlocal nonlinear diffusion problems of $p$-Laplacian type with nonlinear boundary conditions posed in random walk spaces. These spaces are of great generality and include, among others, weighted discrete graphs and $\mathbb{R}^N$ with a random walk induced by a nonsingular kernel. We also study nonlinear dynamical boundary conditions. The generality of the nonlinearities considered allow us to cover the nonlocal counterparts of a large scope of local diffusion problems: Stefan problems, Hele-Shaw problems, diffusion in porous media problems, obstacle problems, and more. The basis for the study is nonlinear semigroup theory.

References:


Anisotropic Chan-Vese segmentation

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Abstract: In this talk I will recall the classical two-phases (or multiphase) Chan-Vese model on piecewise constant segmentation (see [1], [2], respectively) and the relationship with solutions to the Rudin-Osher-Fatemi denoising model ([3]) stated in [4]. I will present some new recent results concerning an anisotropic variant of the aforementioned models.

First, in the 2-phases Chan-Vese case, I will present existence of minimizers and how they are related to the (anisotropic) Rudin-Osher-Fatemi denoising model. Moreover, in the natural case of a piecewise constant on rectangles image (PCR function in short), there exists a minimizer of the Chan-Vese functional which is also piecewise constant on rectangles over the same grid that the one defined by the original image.

In the multiphase case, I will show that minimizers of the Chan-Vese multiphase functional also share this property in the case that the initial image is a PCR function. I will also present a multiphase and anisotropic version of the Truncated ROF algorithm, and I will compare the solutions given by this algorithm with minimizers of the multiphase anisotropic Chan-Vese functional.

Finally, I will show some algorithms for the computation of the minimizers and some numerical experiments related to the results presented.

References:
Weak solutions to the total variation flow in metric measure spaces

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Abstract:
The total variation flow in Euclidean spaces is formally the equation
\[ u_t = \text{div} \left( \frac{Du}{|Du|} \right) \quad \text{in} \; \Omega \times (0, \infty), \tag{*} \]
and it is one of the most popular tools in image processing. From the mathematical point of view, it has been studied in two principal ways: using weak solutions, defined using the classical theory of maximal monotone operators, or variational solutions defined through an energy inequality. In the Euclidean setting, the two approaches are complementing and are known to coincide for sufficiently regular domains and initial data.

In this talk, we present some recent results on the Dirichlet and Neumann problems for the total variation flow in metric measure spaces (see [1]). Assuming that the space \((X, d, \nu)\) and the domain \(\Omega\) are regular enough to guarantee existence of traces, we present a unified framework for the study of the TV flow in the metric setting, based on a metric analogue of the definition of weak solutions. Our main tools are a metric version of the Anzellotti pairings, an associated Gauss-Green formula, and a linear differential structure due to Gigli.

We present a characterisation of weak solutions and prove their existence and uniqueness for the Dirichlet and Neumann problems with initial data in \(L^2(\Omega, \nu)\), using the classical theory of maximal monotone operators. We prove that this notion of solutions is consistent with the variational solutions to the Dirichlet problem (up to the choice of an extension); the Neumann problem has not yet been studied in the metric setting. We also introduce a notion of entropy solutions for the Neumann problem with initial data in \(L^1(\Omega, \nu)\) using the theory of completely accretive operators, prove their existence and uniqueness, and show that they are consistent with weak solutions for \(L^2\) initial data. Furthermore, we show that the weak solutions of the Neumann problem reach the average of the initial data in finite time, and that weak solutions of the homogeneous Dirichlet problem reach zero in finite time. In both cases, we also obtain bounds of the extinction time and the asymptotic profiles.

References:
On the first Robin eigenvalue of the $p$-Laplacian as $p$ goes to 1

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Abstract:
The aim of the talk is to describe the recent results obtained in [1]. In particular, we study the Γ-limit, as $p \to 1$, of the functional

$$J_p(u) = \int_{\Omega} |\nabla u|^p + \beta \int_{\partial \Omega} |u|^p,$$

where $\Omega$ is a smooth bounded open set in $\mathbb{R}^N$, $p > 1$ and $\beta$ is a real number. Among our results, for $\beta > -1$, we derive an isoperimetric inequality for $\Lambda(\Omega, \beta) = \inf_{u \in BV(\Omega), u \not\equiv 0} \frac{|Du|(\Omega) + \min(\beta, 1) \int_{\partial \Omega} |u|}{\int_{\Omega} |u|}$, which is the limit as $p \to 1^+$ of $\lambda(\Omega, p, \beta) = \min_{u \in W^{1,p}(\Omega)} J_p(u)$. We show that among all bounded and smooth open sets with given volume, the ball maximizes $\Lambda(\Omega, \beta)$ when $\beta \in (-1, 0)$ and minimizes $\Lambda(\Omega, \beta)$ when $\beta \in [0, \infty)$.

References:
[1] Della Pietra F., Nitsch C., Oliva F., Trombetti C., On the behaviour of the first eigenvalue of the $p$-Laplacian with Robin boundary conditions as $p$ goes to 1, Advances in Calculus of Variations, in press.
Eigenvalue problems for the p–Laplacian in the critical range
$1 < p < 2$

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Abstract: The second eigenvalue $\lambda_2$ to the Neumann problem,
\[
\begin{aligned}
-\Delta_p u &= \lambda |u|^{p-2} u, & x \in \Omega, \\
|\nabla u|^{p-2} \frac{\partial u}{\partial \nu} &= 0, & x \in \partial \Omega,
\end{aligned}
\]
where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain with outward unitary normal $\nu$, is indeed the first nontrivial eigenvalue. It possesses a variational characterization which is much simpler than its Dirichlet counterpart ([2]). However, obtaining such a variational expression is by means straightforward in the singular case $1 < p < 2$. In this talk we are providing a natural approach to show this formula. In addition, the results can be employed both to address some nonlinear versions of the Neumann problem as well as the Steklov eigenvalue problem ([1]).

References:


Existence and uniqueness for the inhomogeneous 1-Laplace evolution equation revisited

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Abstract: In this talk we deal with an inhomogeneous parabolic Dirichlet problem involving the 1-Laplacian operator in a bounded domain $\Omega$. We show the existence of a unique solution when initial data belong to $L^2(\Omega)$ and sources to $L^1(0,T;L^2(\Omega))$ for every $T > 0$. As a consequence, global existence and uniqueness for sources in $L^1_{loc}(0,\infty;L^2(\Omega))$ is obtained.

The homogeneous problem was first solved in [1] using nonlinear semigroup theory. Authors introduced a concept of solution and proved existence and uniqueness. A new viewpoint has been developed in [2]. Its main feature is that it uses a purely variational approach to deal with time dependent problems, yielding the existence of global parabolic minimizers.

The inhomogeneous setting was first considered in [4] for data belonging to $L^2(0,T;L^2(\Omega))$, also by means of nonlinear semigroups. In [3] authors went a little further and handled data in $L^1(0,T;L^2(\Omega))$. Nevertheless, there is a mistake in their argument. Our analysis retrieves the same results in a correct and complete way.

References:

MS08. Progress on time integrators for ODE
Analyzing and extending existing classes of methods by means of the theoretical framework of General Linear Methods

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Abstract: In this work we use the theoretical framework of General Linear Methods (GLMs) to analyze and generalize some well-known classes of methods. In particular we first focus on multistep methods such as Backward Differentiation Formulae (BDF) and Modified Extended BDF [2], to derive the class of Generalized Linear Multistep Methods [3]. Then, we focus on multistage methods, extending the class of Runge-Kutta methods by means of Self Starting General Linear Methods [4]. This analysis indicates that the new methods may have better accuracy and stability properties, including, for example, wider stability regions in the case of explicit methods, or stage order greater than one for singly diagonally implicit methods.

The possibility of identifying good families of methods with a larger number of degrees of freedom can also have implications in other applications, such as the time discretization of partial differential equations. For example, Self Starting GLMs allow the determination of new efficient and highly stable Implicit-Explicit methods [1].

Finally, we report numerical experiments which confirm that the proposed methods are competitive with original ones and can have better performance on nonstiff, mildly stiff and stiff problems.

Part of this work is joint with Zdzislaw Jackiewicz, Arizona State University (USA), and Sebastiano Boscarino, University of Catania (Italy).

References:
Low order, low storage exponentially fitted explicit Runge-Kutta methods

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Abstract: In this talk we present the class of numerical methods called “low storage exponentially fitted explicit Runge-Kutta” (LSEFRK) schemes with the property of minimum storage requirements for systems with large dimension and whose solution is oscillatory. A study of minimum storage schemes belonging to the van der Houwen and Williamson families with orders $p = 3, 4$ that require only two storage locations is carried out. Two optimal third-order LSEFRK formulae and two optimal fourth-order LSEFRK formulae are deduced taking into account accuracy and stability. Some numerical experiments are presented to show the behaviour of the new low storage EFRK schemes.

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Highly stable explicit numerical methods with Jacobian dependent coefficients for differential problems

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Abstract: The aim of this talk is to employ techniques [1] allowing to modify the coefficients of classic explicit numerical methods, improving their stability properties, for the solution of ordinary differential equations systems of the form:

\[ y' = f(t, y(t)), \quad f : \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}^d, \quad t \in [t_0, T], \]

arising in the semi-discretization in space of partial differential equations. The new methods coefficients depend on the Jacobian of the problem (1).

The mentioned methodologies have been applied to Runge-Kutta methods, and it has been shown that it is possible to make them A-stable or A(\(\theta\))-stable [2]. By combining such techniques with nonstandard finite differences [4], we show that it is possible to make A-stable or A(\(\theta\))-stable parallelizable explicit peer methods [3]. We perform numerical tests to highlight the advantages of the new methods thus obtained, in terms of stability, accuracy and efficiency.

References:

Parallel-in-time splitting-based methods for the solution of parabolic equations

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Abstract:
Splitting time integrators are known to be a suitable tool for the time discretization of spatially discretized parabolic equations. They may consider different partitions of the elliptic operator, such as component-wise dimensional splittings or partitions related to appropriate decompositions of the spatial domain, among others. In both cases, the parallel properties of the resulting algorithms permit to reduce the computational complexity of the solution procedure for the underlying system of ordinary differential equations.

In this work, in order to consider the time variable as a further direction for parallelization, we propose and analyze suitable combinations of the aforementioned splitting time integrators with the well-known parallel-in-time parareal algorithm (cf. [1]). In this framework, both classical alternating direction and domain decomposition splittings will be considered. Numerical experiments illustrating the behaviour of the proposed algorithms will be shown.

References:

Efficient Runge-Kutta-TASE methods for the solution of Stiff problems

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• Luis Rández (randez@unizar.es) IUMA - Univ. de Zaragoza

Abstract:
A new family of Time-Accurate and highly-Stable Explicit (TASE) operators for the numerical solution of stiff Initial Value Problems (IVPs) is proposed. The new TASE operators require the solution of several linear systems, all of them with the same matrix of coefficients (thus we call them Singly TASE operators), which can be more efficient than Bassenne’s family [1, 2] that require the solution of linear systems with k different matrices to get order k. The A–stability properties of explicit Runge-Kutta (RK) schemes with Singly TASE operators with order k ≤ 4 is studied. Schemes with order two, three and four that are nearly strongly A–stable and therefore suitable for stiff problems are obtained. A set of numerical experiments has been conducted to demonstrate the performance of the new methods by comparing with previous Runge-Kutta TASE, SDIRK and Rosenbrock methods.

References:

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Abstract:
Many physical phenomena can be modeled by ordinary and partial differential equations [1]. Solving these equations is a challenge since most of them do not have an analytical solution and usually require numerical methods that discretize the domain of interest [2]. We present an alternative to these methods with the use of neural networks as an approximation function of the analyzed equation. [3]. Two approaches are described. The first substitutes the classical numerical methods in the solution of a fixed problem, with given boundary conditions. The second uses a trained network to predict solutions of ordinary differential equations given new initial conditions and parameters. We apply these approaches to various differential equations that present some degree of difficulty for classical numerical methods. The proposal has the advantage that the neural network, as a universal approximator, can in principle be applied to any problem of this type and the approximate solution can be reached with little data. The fundamental disadvantage is that it is very difficult to standardize the configuration and training parameters of the network to obtain the best results. To date, these parameters must be selected empirically.

References:
Efficient time integration of nonlinear partial differential equations by means of Rosenbrock methods

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Abstract: We avoid as possible the order reduction of Rosenbrock methods when they are applied to nonlinear partial differential equations. For this, we use a suitable choice of boundary values for the internal stages. The implementation is cheap and simple since, at each stage, just some additional terms concerning those boundary values and not the whole grid must be added to what would be the standard method of lines.

References:

Boundary corrections for splitting methods on multi-dimensional PDEs

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Abstract: In this talk we will focus on some splitting methods of ADI-type for the time solution of multi-dimensional parabolic PDEs, when finite differences are applied for their spatial discretization [2, 1]. These methods, due to their relatively low computational cost, can be suitable to integrate high-dimensional problems, as the ones coming from option pricing in Finance, where five or six different interest rates are usually considered as “spatial” variables. However, they suffer from order reduction when time-dependent boundary conditions are imposed [2]. To avoid this, two different boundary corrections are introduced when imposing time-dependent Dirichlet boundary conditions on multi-dimensional PDEs. The first one consists of expanding the ADI-type method to the boundary values while the second correction expands the PDE to the borders. This latter strategy is more computationally expensive than the former, but has the advantage of being independent of the method used for the time integration. Several numerical experiments will be presented in which the recovery of the order will be observed obtaining the one reached with homogeneous boundary conditions. In addition, we will compare the behaviour of the two proposed corrections.

Acknowledgements.- Funding from University of La Laguna and the Spanish Ministry of Science, Innovation and Universities is acknowledged.

References:


Taylor-Fourier integrators

Authors:
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• Joseba Makazaga (joseba.makazaga@ehu.eus) UPV/EHU
• Ander Murua (Ander.Murua@ehu.eus) UPV/EHU

Abstract: In this talk we study Taylor-Fourier integrators for the numerical solution of differential systems

$$\frac{d}{dt}x = Ax + g(x),$$

(1)

where $g$ is a smooth map and $A$ is a real matrix whose eigenvalues are integer multiples of an imaginary number $i\omega$. Based on ideas pointed out in [1], the numerical solution of (1) is constructed by computing a sequence of approximations to the solution of the differential system

$$\frac{d}{dt}y = f(\omega t, y),$$

obtained after making in (1) the change of variables $x(t) = e^{tA}y(t)$. Implementation details and numerical results to show the performance of the methods will be given.

References:

A new insight on positivity and contractivity of Crank-Nicolson scheme

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Abstract:
In this work we obtain necessary and sufficient conditions for numerical positivity and contractivity in the maximum of Crank-Nicolson method when applied to the diffusion equation with Dirichlet boundary conditions. We have obtained bounds for any value of the discretization parameter. In the limit, when the parameter tends to infinity, we recover some known results in the literature [1],[3].

The analysis has been done by using a kind of Chebyshev-like polynomials and it can be extended to the $\theta$-method and other schemes.

References:


MS09. ORTHOGONAL POLYNOMIALS, SPECIAL FUNCTIONS AND APPROXIMATION THEORY
Orthogonal polynomials on the unit circle and functional differential equations

Authors:
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- A. Iserles (A.Iserles@dampt.cam.ac.uk) University of Cambridge

Abstract:
Numerous examples of orthogonal polynomials on the real line are known in an explicit form, but this is not the case for orthogonal polynomials on the unit circle. In this talk we present recent results concerning a far-reaching generalization of the Rogers-Szego polynomials. A generating function of these polynomials obeys a functional differential equation of the pantograph type, and this allows us to deduce the explicit form of these polynomials in terms of q-hypergeometric functions.
Ladder operators and a second–order difference equation for Sobolev–type orthogonal polynomials

Authors:

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• Juan F. Mañas (jmm939@ual.es) Universidad de Almeria
• Juan J. Moreno–Balcázar (balcazar@ual.es) Universidad de Almeria

Abstract:

We consider a general Sobolev–type inner product involving the Hahn difference operator, so this includes the well–known difference operators $\Delta_q$ and $\Delta$ and, as a limit case, the derivative operator. The objective is to construct the ladder operators for the corresponding nonstandard orthogonal polynomials and in this way we deduce the second–order difference equation satisfied by these polynomials. Moreover, we will show that all the functions involved in these constructions can be computed explicitly (see [1]).

References:

Sobolev orthogonal polynomials and spectral methods in boundary value problems

Authors:
• Francisco Marcellán (pacomarc@ing.uc3m.es) Universidad Carlos III de Madrid

Abstract:
In this contribution we study some sequences of polynomials orthogonal with respect to a Sobolev inner product related to the vector of measures \( \{x^2 dx, dx\} \) supported on the interval \([-1, 1]\). They appear in boundary value problems for Ordinary Differential Equations associated with either an harmonic potential (in a second order problem) or Dirichlet conditions (in a fourth order problem). Properties of such polynomials are analyzed as well as its performance in the framework of spectral methods. These polynomials have been introduced in [2]. We will follow the approach to Sobolev orthogonal polynomials given in the survey paper [1].

References:


On Sobolev orthogonal polynomials on the simplex

Authors:
• Misael Marriaga (misael.marriaga@urjc.es) Universidad Rey Juan Carlos

Abstract:
We use the invariance of the simplex $T^2 = \{(x; y) \in \mathbb{R}^2 : 0 \leq x, y, 1 - x - y\}$ under the permutations of $\{x, y, 1 - x - y\}$ to construct and study two-variable orthogonal polynomial systems with respect to several distinct Sobolev inner products defined on $T^2$. These orthogonal polynomials can be constructed from two sequences of univariate orthogonal polynomials. In particular, one of the two univariate sequences of polynomials is orthogonal with respect to a Sobolev inner product and the other is a sequence of classical Jacobi polynomials.
Quasi-birth and death processes

Authors:
- Lidia Fernández (lidiafr@ugr.es) Universidad de Granada
- Manuel D. de la Iglesia () Universidad de Granada

Abstract:
Birth and death processes have many applications and are very useful in modeling. The aim of this paper is to study some models of quasi-birth-and-death (QBD) processes arising from the theory of bivariate orthogonal polynomials. We will see how to perform the spectral analysis in the general setting as well as some results about recurrence and the invariant measure of these processes in terms of the spectral measure supported on some domain in $\mathbb{R}^d$. Afterwards, applications to several examples of bivariate orthogonal polynomials will be presented. We will focus on linear combinations of the Jacobi matrices generated by these polynomials and produce families of either continuous or discrete-time QBD processes. Finally, we show some urn models associated with these QBD processes.
Discrete Appell-Dunkl sequences

Authors:
• Judit Mínguez (judit.minguez@unirioja.es) Universidad de La Rioja

Abstract:
In a similar way that the Appell sequences of polynomials can be extended to the Dunkl context, where the ordinary derivative is replaced by Dunkl operator on the real line, and the exponential function is replaced by the so-called Dunkl kernel (see [1] and [2]), one can expect that the discrete Appell sequences can be extended to the Dunkl context. In this extension, the role of the ordinary translation is played by the Dunkl translation, that is a much more complicate operator.

In this conference, we define discrete Appell-Dunkl sequences of polynomials, and we show some properties and examples such as discrete Bernoulli-Dunkl polynomials (see [3]). We also define the Stirling-Dunkl numbers of first and second kind.

References:


A convergent and asymptotic Laplace method for integrals

Authors:
• Jose L. Lopez (jl.lopez@unavarra.es) Universidad Pública de Navarra
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Abstract:
Watson's lemma and Laplace's method provide asymptotic expansions of Laplace integrals

\[ F(z) := \int_0^\infty e^{-zt}f(t)g(t)dt \]

for large values of the parameter \( z \). They are useful tools in the asymptotic approximation of special functions that have a Laplace integral representation. But in most of the important examples of special functions, the asymptotic expansion derived by means of Watson’s lemma or Laplace’s method is not convergent.

In this work, we combine the modified Laplace’s method introduced in [?] with a logarithmic change of variables to derive asymptotic expansions of \( F(z) \) that are also convergent, accompanied by error bounds.

As illustration, a new convergent and asymptotic expansion of the parabolic cylinder function \( U(a, z) \) for large \( |z| \) is derived.

References:
Best algebraic bounds for ratios of modified Bessel functions

Authors:
• Javier Segura (segurajj@unican.es) Universidad de Cantabria

Abstract:
The best possible algebraic bounds of the form $B(\alpha, \beta, \gamma, x) = (\alpha + \sqrt{\beta^2 + \gamma^2 x^2})/x$ for ratios of modified Bessel functions (both of the first and second kinds) are characterized. The bounds are sharp as $x \to 0^+$ and/or as $x \to +\infty$ and at any given $x_\ast > 0$, and they are the best possible bounds around such $x_\ast$ (a different bound for each value of $x_\ast$). Bounds with maximal accuracy at $0^+$ and $+\infty$ are recovered in the limits $x_\ast \to 0^+$ and $x_\ast \to +\infty$, and for these cases the coefficients have simple expressions. For the case of finite and positive $x_\ast$ we provide uniparametric families of bounds which are close to the optimal bounds and retain their confluence properties.
MS10. SUCCESS STORIES BETWEEN ACADEMIA AND INDUSTRY
AT CITMAGA
Mathematical modeling and numerical simulation in SisAl project, an innovative pilot for silicon production

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• Dolores Gómez (mdolores.gomez@usc.es) CITMAga, Dpto. Matemática Aplicada, Universidade de Santiago de Compostela

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Abstract:

SisAl Pilot is an Horizon 2020 funded project coordinated by Norwegian University of Science and Technology (NTNU) which comprises 22 partners from 9 countries. The main objective of this project is to demonstrate a patented novel industrial process to produce silicon. The actual carbothermic Submerged Arc Furnace (SAF) process is replaced by a far more environmentally and economically sustainable alternative: the aluminothermic reduction of quartz, which allows to use secondary raw materials such as aluminium (Al) EoL scrap and dross, instead of carbon reductant used today. To attain this goal, different types of furnaces are being analyzed. In this talk we will focus on modelling and simulation of induction and rotary furnaces. Depending on the furnace, the simulations require to study several physical processes strongly coupled: heat transfer, multiphase fluid dynamics, electromagnetism, melting processes and chemical reactions. Thus, the challenge is to carry out numerical simulations based on these models that can support the experimental trials in plant of the industrial partners.
Optimization of an industrial paper air-drying line using a reduced model derived from CFD

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Abstract: A numerical tool for the fast prediction of an industrial air-drying line of resin-impregnated paper is presented. The reduced model, obtained from local mass and energy balance equations for the paper sheet, was built using results both from time-consuming three-dimensional fluid dynamic simulations of the industrial furnaces, and from drying thermo-gravimetric tests. The validated numerical tool is capable of predicting, with a reduced computational time, the evolution of the paper weight and temperature along the line for any given combination of the numerous production parameters: paper feed rate in the line, furnaces air temperatures and mass flows, among others. The surrogate model is then coupled to an optimization tool to be used on the inverse design of the process, that is, on the selection of the best production parameters for prescribed production conditions, making the model a useful tool in the framework of the increasingly relevant role of digital twins in industry.

References:
Optimal biddings for a renewable energy production plant with a storage system

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Abstract: One of the challenges in the industrial energy sector is the possibility that renewable energy plants can operate in the energy market in an autonomous way without depending on other types of sources to comply with the energy commitments previously acquired. The delivery of this energy is often required to be at constant power which makes the task even more difficult to accomplish. In order to achieve these objectives and to mitigate the uncertainty in energy generation caused by weather changes, it is proposed to incorporate energy storage systems based on lithium-ion batteries [1]. During the talk, optimization problems aimed at optimizing a plant with these characteristics will be discussed. In order to incorporate the state of charge and the state of health of the battery, a digital twin based on electrochemical models [2, 3] is used. The resolution of these problems provides a tool that allows both the dimensioning of the storage system and optimized energy biddings.

References:

Computation of resonances in underwater acoustics using Perfectly Matched Layers

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Abstract: Nowadays, acoustic techniques implemented in sonar equipment are widely used in industrial applications. They provide a common methodology for monitoring and detecting the underwater environment, the profile of the seabed, or the presence of biological elements. In any of these cases, the wave propagation throughout the environment is settled in unbounded domains, where no natural boundaries are placed close to the acoustic source.

The characterization of the acoustic properties of the layered media involved on these underwater problems can be analyzed by means of the computation of resonances of the vibro-acoustic problems stated on these unbounded domains as it has been already illustrated in other layered media (see, for instance, a similar industrial application in [2]). In the present work, to deal with the challenge of computing resonances related to problems stated in unbounded domains, Perfectly Matched Layers (PML) method [1] is used to truncate the unbounded physical domain and enable the use of a standard finite element.

Since the original underwater acoustic problem must be coupled with the PML governing equations, the spectrum of the original problem is modified and the physical relevant resonances are numerically polluted by the use of the PML and its configuration (absorption profile, thickness, location, etc.). The finite element approximation of resonances will be computed in different underwater scenarios, highlighting the difference between physical and polluted resonances presented in the approximated spectrum.

References:


Physically Based Reduced Order Battery Models Including Heterogeneous Degradation for Real-Time Control Applications

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Abstract: Lithium-ion batteries represent one of the most widely used energy storage devices in both mobile and stationary applications. Its correct operation depends largely on the so-called Battery Management Systems (BMS) that use mathematical models to be able to predict in real time the state of charge, the state of health and the state of function of the battery in order to make a decision. Although equivalent circuit models [1] are often used for this purpose, nowadays advanced BMS incorporate more complex electrochemical models [1, 2, 4]. Since these models need to be solved faster than in real time, it is necessary to apply specific order reduction techniques [3]. In this talk, various mathematical models and numerical techniques compatible with the simulation of these devices in real-time applications will be discussed.

References:
Real-time Boundary Heat Flux Estimation in Continuous Casting Molds Using Data Assimilation

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- Patricia Barral (patricia.barral@usc.es) Universidade de Santiago de Compostela

Abstract: In continuous casting of steel, the most critical component is the mold. In the mold, the steel begins its solidification, and several complex physical phenomena happen. To ensure a proper control of the process, it is necessary to know how the steel is behaving inside the mold. However, it is not possible to make measurements inside the solidifying steel, and the only available data are pointwise temperature measurements in the interior of the mold plates. To provide a tool for the proper control of the process, we developed a methodology for the real-time estimation of the heat flux at the steel-mold interface given the temperature measurements. With this tool, we allow the caster operator to quickly detect any malfunctioning in the casting, increasing the safety and productivity of continuous casters.

References:

Modelling, numerical simulation and optimal control problems related to regasification plants

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Abstract:
Nowadays management and supplying energy to customers are the most important part of energy security. Among the different types of energy sources, natural gas is one of the most sustainable and common source which can be stored in form of liquefied natural gas (LNG). The aim of liquefying natural gas is to occupy much less volume than natural gas. Thus, long distance transportation by ships and trucks is more economical than building and using gas pipelines. However, regasification plants are needed and therefore, simulation and optimization of these plants are important mathematical tools for design and operation purposes (see, for instance, [1, 2]). Each device of the plant is mathematically modelled and considered as a box with some inputs and outputs. These boxes are connected through edges which transfer the information from one box to another and so the whole regasification plant is modelled by Differential Algebraic Equations (DAEs). In addition, to simulate and optimize the regasification plant, a Mixed Integer Quadratic Program (MIQP) is formulated that is able to make decisions to turn on and off some devices (i.e., compressors and pumps) by minimizing a cost function. The final product of this research is a Python simulation and optimization package which can be used for planning and optimizing regasification plants based on minimizing the electricity costs while guaranteeing the security of supply and the preservation of technical constraints. Finally, several specific cases are solved and the results are discussed.

References:
Thermo-electromagnetic-mechanical simulation of an electric upsetting process in automotive industry based on Lagrangian formulations

Authors:
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Abstract: The objective of this work is to describe the mathematical models and numerical techniques developed to simulate the electric upsetting process used by the Spanish company CIE Galfor in the manufacturing of components for the automotive market. The research has been developed in collaboration with this company through a research project granted by the Centre for the Development of Industrial Technology (CDTI) and signed between the company CIE Galfor and the Technological Institute for Industrial Mathematics (Itmati) (nowadays, integrated in CITMAga).

Electric upsetting is a multiphysics process which involves large deformations and a strong and fast change in the temperature due to the electric heat generated in the workpiece. In this work we will describe a thermo-electromagnetic-mechanical model based on Lagrangian formulations which allow us to compute the power generated in an axisymmetric workpiece and the temperature and the deformation of the piece at any time and under different operation conditions. We will present the mathematical models and the coupling factors between them, the Lagrangian description for the coupled problem and the numerical tools used for the discretization and for dealing with the strong non-linearities. Finally, we will show some academic examples for validating the numerical techniques and results corresponding to the electric-upsetting process implemented in the industrial plant.
MS11. Reduced Order Modeling applied to Architecture and Engineering
Reduced Order models and methods. Applications to transition spaces in buildings

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- Samuele Rubino (samuele@us.es) Universidad de Sevilla

Abstract: In this talk we present the study of the influence of courtyards on the energy efficiency of buildings. Due to the complexity of the problem, the classical numerical methods of solving the mathematical equations require a fairly long time to solve, which makes them unaffordable and therefore it is necessary to use methods that allow us to obtain results in an acceptable time. In this case we use reduced order methods from two points of view, on the one hand we study the behaviour using the equations of the model with geometrical and physical parameters, designing for this purpose a posteriori error estimators. On the other hand, we make a study based on experimental data that allows us to obtain the thermal behaviour of a patio using reduced order methods, obtaining results with a lower error than the commercial programs commonly used in this field.
Numerical resolution of problems with dominant convection by stabilized methods based on data

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Abstract:
In this work, we present a method to optimise the coefficients that appear in the stabilisation terms for problems with dominant convection, as a solution to least squares approximation problems.

In the Galerkin discretisation of partial differential equations, instabilities appear when the PDEs include terms of different order of derivation and the discretisation parameters are not small enough, as occurs in convection dominant problems.

Different stabilized methods have been designed, all of them consisting of adding additional terms to the Galerkin formulation with the aim of controlling one or several operator terms that appear in the equations (Cf. [1]).

The method presented here includes an off-line step in which the stabilised coefficients are determined by quadratic error minimisation, as functions of the relevant non-dimensional parameters of the discretised convection-diffusion equation. These coefficients are computed at the nodes of a grid whose axis are these parameters. The actual stabilised coefficients are interpolated in the on-line step from the values computed at the grid nodes. In this way the computational time required by this method is largely reduced.

Our results show that the already built stabilised methods provide nearly optimal error levels, although our technique provides some improvement for high-order finite element interpolation.

References:
Reduced Basis Modeling for LPS pressure VMS-Smagorinsky model

Authors:
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Abstract: In this work we present the numerical analysis of a Reduced Basis VMS-Smagorinsky model with local projection stabilization (LPS) on the pressure. We construct the reduced velocity space by two different strategies, by considering or not the enrichment of the reduced velocity space with the so-called inner pressure supremizer. We present the development of an a posteriori error estimator for the snapshot selection through a Greedy algorithm, based on the Brezzi-Rappaz-Raviart (BRR) theory. Moreover, the Empirical Interpolation Method (EIM) is considered for the approximation of the non-linear terms. Finally, we present some numerical tests in which we show an speedup on the computation of the reduced basis problem with the LPS pressure stabilisation, with respect to the method using pressure supremizers.

References:

Residual-based data-driven Variational Multiscale Reduced Order Models

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Abstract:
ROMs for fluid flows have been used to successfully minimize the computational cost of scientific and engineering applications dominated by a small number of recurring dominant spatial structures. For under-resolved simulations, using a relatively small number of ROM basis functions often yields an inaccurate approximation. Since numerical efficiency is one of the big advantages of the ROM, we want to increase the numerical accuracy while preserving the computational efficiency. Thus, we add a low-dimensional closure term \( \text{Closure}(a) \) to the standard Galerkin ROM and solve the following closed ROM,

\[
\dot{a} = F(a) + \text{Closure}(a),
\]

where the \( \text{Closure}(a) \) term models the interaction between the unresolved ROM basis functions \( \{\varphi_{r+1}, \ldots, \varphi_d\} \) and the resolved ROM basis functions \( \{\varphi_1, \ldots, \varphi_r\} \).

In this talk, we propose a consistent data-driven (D2) variational multiscale (VMS) reduced order model (ROM) framework to increase the ROM accuracy at a modest computational cost for under-resolved regimes. To construct the new consistent D2-VMS-ROM, we need to model the closure term by using the residual term as:

\[
\text{Closure}(a) \approx \hat{A} \text{Res}(a).
\]

References:
An approach to Reduced Basis Large Eddy Simulation turbulence models based upon Kolmogorov’s equilibrium turbulence theory

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- Macarena Gómez Mármol (macarena@us.es) Dpto. EDAN, Universidad de Sevilla

Abstract: In this talk we will afford the reduced basis approximation of Smagorinsky’s turbulence model. With basis on Kolmogorov’s turbulence theory in statistical equilibrium (Cf. [1]), we will derive an error estimator. This estimator will measure the deviation of a trial solution with respect to the theoretical $k^{-5/3}$ spectrum of a velocity field corresponding to well-developed turbulence.

This error estimation procedure has the advantage of applying to any kind of numerical discretisation, and to any physical time at which the turbulence is in statistical equilibrium. This allows to overcome the technical difficulties related to the building of error estimation for Reduced Basis discretisation of non-linear Partial Differential Equations [2].

We will present some results for academic, yet meaningful, flows, that exhibit the good features of this approach.

References:


Model Order Reduction: A geometric approach

Authors:
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Abstract:
The aim of this talk is to ask the question if it is possible, for a given dynamical system defined by a vector field over a finite dimensional inner product space, to construct a reduced order model over a finite dimensional manifold. In order to give a positive answer to this question, we prove that if the manifold under consideration is an immersed submanifold of the vector space, considered as ambient manifold, then it is possible to construct explicitly a reduced order vector field over this submanifold. In particular, we found that the reduced order vector field satisfies the variational principle of Dirac-Frenkel and that we can formulate the Proper Orthogonal Decomposition under this framework. Finally, we propose a local point estimator of the time-dependent error between the original vector field and the reduced order one.
Certified Reduced Order Method for Parametrized Allen-Cahn Equation

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Abstract: We are interested in physical problems that can be modeled by partial differential equations having the form of gradient flows. Such models come from energy gradient flows describe dynamics driven by a free energy, long used in many fields of science and engineering, particularly in materials science and fluid dynamics, see, e.g., [1, 2] and the references therein. Typical examples include the Cahn-Hilliard (CH) and Allen-Cahn (AC) equations for multi-phase flows, for which the evolution PDE system is resulted from the energetic variation of the action functional of the total free energy in different Sobolev spaces.

In this work we propose to consider a parametrized Allen-Cahn equation and to propose a certified model reduction strategy to approximate it. The talk will be organized as follows. First, we consider the Allen-Cahn equation, construct a full-order model and its corresponding reduced-order model. We also establish stability analysis and error estimates for two fully discretized schemes with finite Element approximation and reduced order element approximation in space respectively. Then, we consider the parameterized Allen-Cahn equation and carry out the corresponding reduced-order model. Finally we establish stability and error analysis and we assess the previous convergence results with some numerical examples.

References:


Efficient time integration of semilinear wave problems with time-dependent boundary values using splitting methods

Authors:
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- Ana Portillo (ana.portillo@uva.es) IMUVA, Universidad de Valladolid

Abstract: An initial boundary-value problem associated to a semilinear wave equation with time dependent boundary values may be approximated using the method of lines. Here we deal with the case in which the time integration is achieved by means of an explicit time method obtained from an arbitrarily high-order splitting scheme. We propose a novel technique to incorporate the boundary values that is more accurate than the one obtained in the standard way, which is clearly seen in the numerical experiments. We prove the consistency and convergence, with the same order of the splitting method, of the full discretization carried out with this technique.

References:

A high-order Lagrange–Galerkin method for compressible flows

Authors:
• Manuel Colera (m.colera@upm.es) Universidad Politécnica de Madrid
• Jaime Carpio (jaime.carpio@upm.es) Universidad Politécnica de Madrid
• Rodolfo Bermejo (rodolfo.bermejo@upm.es) Universidad Politécnica de Madrid

Abstract:
We present a novel Lagrangian–Eulerian scheme for the resolution of two-dimensional compressible and inviscid flows. The scheme considers arbitrary-order continuous space discretizations on unstructured triangular meshes, as well as arbitrary-order implicit–explicit Runge–Kutta time marching schemes. The method preserves mass, momentum and total energy as long as the integrals in the formulation are computed exactly. The recent model proposed by Brenner [1] for viscous flows is employed to define the operators needed to stabilize the continuous Galerkin formulation. The method has been tested on several benchmark problems using a fourth-order time-marching formula and up to fifth-order elements, showing good accuracy both for smooth and discontinuous solutions.

Acknowledgements
This research has been partially funded by project PGC-2018-097565-BI00 from the “Ministerio de Ciencia, Innovación y Universidades” of Spain (MCIU) and the European Regional Development Fund, and by the FPU16/05509 grant to M. Colera from MCIU.

References:

Linearly implicit splitting methods for semilinear singularly perturbed convection-diffusion systems

Authors:
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• Juan Carlos Jorge (jcjorge@unavarra.es) Universidad Pública de Navarra

Abstract: In this talk we propose and analyze a numerical method for solving 1D semilinear parabolic singularly perturbed systems of convection-diffusion type. In the case of having small diffusion parameters with different orders of magnitude overlapping boundary layers appear close to the outflow boundary. Our proposal combines a linearized version of the fractional implicit Euler method together with a splitting by components, to discretize in time, and the upwind finite difference scheme on an appropriate piecewise uniform mesh, to discretize in space. It is proven and tested that the proposed numerical algorithm is uniformly convergent.
Serverless computing architecture for data processing and detecting anomalies in the ESA Mars Express MARSIS instrument

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Abstract: This work introduces a serverless modular architecture for processing data from the MARSIS instrument (ESA Mars Express Mission). In particular, anomalies are detected in the obtained ionograms once the mission data appears in the repository (almost real time), helping the research team with this preliminary classification.
A posteriori error estimation for an EG method in fluid mechanics

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Abstract: In the recent work [1], the authors present and analyze a new stable enriched Galerkin method for the Stokes problem with a non-homogeneous boundary condition of Dirichlet type. In case of a simplicial mesh, the pressure is approximated by piecewise constants and the velocity is approximated by continuous piecewise polynomials of degree one plus piecewise constants. This pair of spaces satisfies a uniform inf-sup condition (see [1]) in two and three dimensions. The a priori error analysis shows that the method is of order one. Moreover, this new scheme can be used to solve large problems since it requires less degrees of freedom than standard first order discontinuous Galerkin methods and it is easier to implement.

In this talk, we will present the a posteriori error analysis of the method proposed in [1]. We will describe error indicators of residual type and we will study its reliability and efficiency. In both cases, the results are optimal. We will also show some numerical results that confirm the good properties of the method.

References:


Numerical methods for singularly perturbed reaction-diffusion problems with non-smooth data

Authors:

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• Eugene O’Riordan (eugene.oriordan@dcu.ie) School of Mathematical Sciences, Dublin City University, Ireland

Abstract: In this talk singularly perturbed reaction-diffusion parabolic problems with non-smooth initial or boundary data are considered. This is a bi-singular problem class where a classical singularity due to the non-smooth data is entwined with the singular nature of the differential operator when the diffusion parameter takes arbitrary small values. In order to generate a parameter-uniform approximation to the solution, finite difference schemes on appropriate meshes of Shishkin type condensing in the layers regions are considered. Error estimates in the maximum norm are established proving the uniform convergence of the numerical approximations constructed in the talk. Numerical results for several examples are presented showing the sharpness of the established theoretical error bounds.
MS13. Efficient solvers for large sparse linear systems
Solvers for Multiphysics Problems in Brain Biomechanics

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- Miroslav Kuchta (miroslav@simula.no) Simula Research Laboratory
- Kent-Andre Mardal (kent-and@math.uio.no) University of Oslo
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Abstract:
We are interested in reliable simulations of some biophysical processes in the brain, such as blood flow and metabolic waste clearance. Modeling those processes results in interface-driven multiphysics problems that can be coupled across dimensions. However, the complexity of the interface coupling often deteriorates the performance of standard methods to finding the numerical solution. Therefore, we derive preconditioners and solution techniques which target specifically such multiphysics problems for order optimal solving performance.

The coupling that enforces constraints on the interface can primarily be imposed in two ways – with or without a Lagrange multiplier. In the former, the well-posedness of the system of equations is given in fractional Sobolev spaces weighted by material parameters. Therefore, the robust preconditioners for the interface problems are represented as a sum of fractional Laplacians that can include both negative and positive fractionalities. To handle fractional operators numerically, we implement methods based on the rational approximation. For the systems without a Lagrange multiplier, we focus on solvers based on algebraic multigrid method with custom smoothers that preserve the coupling information on each coarse level. We prove that, for the two-level setting, we obtain convergence that is independent of the mesh and material parameters.

In both approaches, we show parameter-independence and scalability with regards to number of the degrees of freedom of the system. This is demonstrated on numerical examples of mixed-dimensional problems on realistic geometries, such as 3D-1D model of flow in vascularized brain tissue.
A posteriori error estimates in finite element method by preconditioning

Authors:
- Yuwen Li (yuwli@psu.edu) Penn State, USA
- Ludmil Zikatanov (ludmil@psu.edu) Penn State

Abstract: We present a framework that relates preconditioning with a posteriori error estimates in finite element methods. In particular, we use standard tools in subspace correction methods to obtain reliable and efficient error estimators. As a simple example, we recover the classical residual error estimators for the second order elliptic equations as well as present some new estimators for systems of PDEs.
Nonlinear solver acceleration based on machine learning applied to multiphase porous media flow

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Abstract: We present a machine learning strategy for accelerating the nonlinear solver convergence for multiphase porous media flow problems. The presented approach dynamically controls an acceleration method based on numerical relaxation. The goal of the machine learning acceleration is to reduce the number of iterations required by the nonlinear solver by adjusting the value of the relaxation factor to the complexity/physics of the system. A set of dimensionless parameters is used to train and control the machine learning. In this way, a simple two-dimensional layered reservoir can be used for training while still exploring a large portion of the dimensionless parameter space.

We demonstrate that the presented technique dramatically reduces the number of nonlinear iterations without sacrificing the quality of the results, even for models that are far more complex than the training case. The average reduction in the number of nonlinear iterations obtained due to the presented method is 24.

The method presented here provides an easy way to deal with nonlinear system of equations that does not necessitate as much effort as a custom nonlinear solver while producing outstanding results. We believe that the machine learning acceleration is not limited to the multiphase porous media flow but extendable to any other system that can be studied based on dimensionless numbers, and that a relaxation technique can be used to stabilize the nonlinear solver.

References:
Monolithic Multigrid for a Reduced-Quadrature Discretization of Poroelasticity

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Abstract:
Advanced finite-element discretizations and preconditioners for models of poroelasticity have attracted significant attention in recent years. The equations of poroelasticity offer significant challenges in both areas, due to the potentially strong coupling between unknowns in the system, saddle-point structure, and the need to account for wide ranges of parameter values, including limiting behavior such as incompressible elasticity. The work presented in this talk was motivated by an attempt to develop monolithic multigrid preconditioners for a novel P1-P0-RT-stabilized discretization developed in [1],[2]; we show here why this is a difficult task and, as a result, we modify the discretization through the use of a reduced quadrature approximation, yielding a more “solver-friendly” discretization. Local Fourier analysis is used to optimize parameters in the resulting monolithic multigrid method, allowing a fair comparison between the performance and costs of methods based on Vanka and Braess-Sarazin relaxation. Numerical results are presented to validate the LFA predictions and demonstrate efficiency of the algorithms. Finally, a comparison to existing block-factorization preconditioners is also given.

References:
An efficient solver for IGA discretizations of Biot’s equations

Authors:
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• Carmen Rodrigo (carmenr@unizar.es) University of Zaragoza

Abstract:
Biot’s model describes the interaction between the deformation of a poroelastic material and the fluid flow inside of it. This model is formulated as a system of partial differential equations (PDEs) and its numerical simulation is crucial for real applications. We consider the discretization of the problem by isogeometric analysis (IGA) [1], which is a numerical technique based on the use of spline-type basis functions for the numerical solution of PDEs. One of its advantages over the classical finite element methods (FEM) is the fact that the same functions can be used for both the numerical approximation of the solution and the construction of the computational domain by applying the isoparametric approach. In addition, isogeometric discretizations allow us to provide approximation spaces with higher order of global smoothness. The discretization of the problem leads to large and sparse systems of algebraic equations. The solution of these systems is the bottleneck in the numerical simulation of real poroelastic problems. Hence, the efficient solution of these systems is a key point. For this purpose, we focus our search of efficient solvers for Biot’s equations into the group of monolithic or fully coupled methods. In this work, we propose a monolithic multigrid method that uses an inexact version of the fixed-stress split method as smoother [2]. This version consists of applying a few iterations of additive Schwarz methods during the decoupled smoothing process instead of direct solvers. In order to design this approach, a local Fourier analysis (LFA) will be performed. This theoretical analysis is based on the Fourier transform and provides us a useful tool for the design of our solver. Finally, some numerical experiments will be shown in order to demonstrate the efficiency of the proposed solver.

References:
A geometric multigrid solver for the Biot problem on logically rectangular grids

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- Carmen Rodrigo (carmenr@unizar.es) University of Zaragoza
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Abstract:

Poroelasticity models study the interaction between the fluid flow and solid deformation within a porous medium. This is of great interest in many areas such as geomechanics, hydrology or biomechanics. A general three-dimensional mathematical formulation was established by Maurice Biot for this kind of problems [1]. Real applications rely on numerical simulation, leading to an intensive research for efficient discretizations and solvers for the algebraic system derived from Biot’s model.

In this work, we consider a discretization of the mentioned model based on the recently introduced multipoint stress-multipoint flux mixed finite element method [2]. This method is locally conservative, can be formulated on simplicial and quadrilateral meshes, and also accurately handles discontinuous full tensor permeabilities and Lamé coefficients, which is the common case found in subsurface flows.

The high computational cost required to solve the large systems of algebraic equations derived from Biot’s discrete scheme makes the design of an efficient solver a crucial aspect in numerical simulation. Monolithic methods and iterative coupling methods are two important approaches for solving the resulting systems. Focusing on the first ones, we propose a geometric multigrid method for the aforementioned discretization scheme on logically rectangular meshes. This choice is motivated by the fact that these meshes take advantage of recent computer architectures, improving the overall performance when structured data is used.

Finally, we present numerical results in order to show the robustness of the proposed solver.

References:


MS14. Nonlinear Analysis in Partial Differential Equations
Elliptic equations with subquadratic growth in the gradient

Authors:
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• Pedro J. Martínez-Aparicio (pedroj.ma@ual.es) Universidad de Almería

Abstract:
In this talk we analyze the following boundary value problem posed in a bounded domain $\Omega \subset \mathbb{R}^N$ with smooth enough boundary $\partial \Omega$

$$
\begin{cases}
-\Delta u = \lambda u + g(x,u)|\nabla u|^q + f(x) & \text{in } \Omega, \\
u = 0 & \text{on } \partial \Omega,
\end{cases}
(P_\lambda)
$$

for every $\lambda \in \mathbb{R}$. Here $1 < q \leq 2$, $f$ belongs to a suitable Lebesgue space and $g$ is a Carathéodory function.

Depending on the behavior of the function $g$ we show existence of solution, nonexistence, multiplicity, homogenization and/or bifurcation.

All the results that we expose are contained in [1, 2, 3].

References:
Fractional equations with nonlocal “gradient terms”

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- Antonio J. Fernández (antonio.fernandez@icmat.es) Instituto de Ciencias Matemáticas, Spain
- Tommaso Leonori (tommaso.leonori@uniroma1.it) “Sapienza” Università di Roma, Italy
- Abdelbadie Younes (abdelbadieyounes@gmail.com) Université Abou Bakr Belkaïd, Algeria

Abstract: We discuss existence and non-existence results for deterministic Kardar–Parisi–Zhang type equations involving non-local “gradient terms”. More precisely, let $\Omega \subset \mathbb{R}^N$, $N \geq 2$, be a bounded domain with boundary $\partial \Omega$ of class $C^2$. For $s \in (0,1)$, we consider problems of the form

$$\begin{cases}
(-\Delta)^s u = \mu(x)|\mathbb{D}(u)|^q + \lambda f(x), & \text{in } \Omega, \\
u = 0, & \text{in } \mathbb{R}^N \setminus \Omega,
\end{cases}$$

where $q > 1$ and $\lambda > 0$ are real parameters, $f$ belongs to a suitable Lebesgue space, $\mu \in L^\infty(\Omega)$ and $\mathbb{D}$ represents a nonlocal “gradient term”. Depending on the size of $\lambda > 0$, we derive existence and non-existence results.

References:

Qualitative properties on the p-Stokes vectorial system

Authors:
- Rafael López-Soriano (ralopez@math.uc3m.es) Universidad Carlos III de Madrid

Abstract:
In this talk we shall study qualitative properties of a p-Stokes type system, namely

\[-\Delta_p u = -\text{div}(|Du|^{p-2}Du) = f(x, u) \text{ in } \Omega,\]

where $\Delta_p$ is the $p$-Laplacian vectorial operator. More precisely, under suitable assumptions on the domain $\Omega$ and the vector field $f$, it will be deduced that system solutions are symmetric and monotone. Our main results are derived from a vector version of the weak and strong comparison principles, which enable to apply the moving-planes technique for systems. These results are based on a joint work with Luigi Montoro and Berardino Sciunzi (Università della Calabria)

References:

Elliptic problems with mixed boundary data for the spectral fractional Laplacian

Authors:
• Eduardo Colorado (ecolorad@math.uc3m.es) Universidad Carlos III de Madrid

Abstract:
In this talk we will show some results about a concave-convex problem with mixed Dirichlet-Neumann boundary data for the spectral fractional Laplacian

\[
\begin{cases}
(-\Delta)^s u = \lambda u^q + u^{p-1} u, & u > 0 \text{ in } \Omega, \\
u = 0 & \text{on } \Sigma_D \subset \partial \Omega, \\
\frac{\partial u}{\partial n} = 0 & \text{on } \Sigma_N \subset \partial \Omega,
\end{cases}
\]

where \( \Omega \) is a bounded domain in \( \mathbb{R}^N \), \( 0 < \lambda \), \( 1/2 < s < 1 \), \( 2s < N \), and \( 0 < q < 1 < p < \frac{N+2s}{N-2s} \). In particular, we will show existence, apriori uniform \( L^\infty \)-bounds and global multiplicity of solutions.

The results are collected in the following references.

References:
MS15. **Industrial Mathematics at the Centre de Recerca Matemàtica**
Developing mathematical models for industrial adsorption columns

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- Alba Cabrera-Codony (alba.cabrera@udg.edu) Universitat de Girona

Abstract:
Tackling environmental challenges is this generation’s defining task (EC Green Deal 2020). One such challenge, the goal of holding global warming “to well below 2°C” (IPCC 2018; Paris Agreement 2015), can now only be achieved through the extraction of atmospheric greenhouse gases in tandem with emission reductions and a range of other measures.

One of the most widely used techniques for removing environmental contaminants is via adsorption columns, where a fluid is passed through a porous matrix which selectively removes certain components. Column sorption is used in a wide range of processes, such as the removal of emerging contaminants, greenhouse gases, volatile organic compounds, dyes and salts. It may be applied to both liquids and gases.

The primary focus of this talk concerns the development, analysis and application of mathematical models for the removal of contaminants by the process of column sorption. The mathematical description of column sorption involves coupling advection-diffusion equations to sink (removal) equations, all applied over an evolving domain. The research topic falls within the broad category of moving boundary problems (MBPs). The ultimate goal of the study is to develop an understanding of the column sorption process which may then be used to design future practical equipment.

A novel (in the field of adsorption at least) approximate analytical solution is developed and compared against numerical and experimental data for a range of contaminants, including CO2. The results show that previous accepted models contain errors. Excellent agreement is demonstrated with certain data: discrepancies with other data sets indicate different adsorption methods dominate with different systems.
Modeling of scale-free marked point processes for hazard assessment

Authors:
• Jordi Baró (jbaro@crm.cat) Centre de Recerca Matemàtica

Abstract: The statistical modeling, nowcasting and forecasting of extreme events posing social or ecological hazards represents a common fundamental asset for risk prevention and mitigation strategies. Due to its potential impact to both public services and the private interests it is contemplated as one of the major challenges for the next decades in the UN’s "Sendai Framework for Disaster Risk Reduction 2015-2030". Extreme-event statistics are often described in terms of stochastic point-processes, marked by a measure of size, or magnitude, which contains physical information and can be linked to its potential hazard. Such magnitudes are often observed to attain a fat-tailed distribution, well modeled as power-laws with low exponent values, or even governed by a few massive events, hindering the estimation of uncertainties in the associated hazards. Theories from out-of-equilibrium statistical physics and chaos [1] predict such black-swan and dragon-king statistics [2] as the consequence of adaptation and self-organization mechanisms leading the system to a critical state, or the edge-of-chaos, where an initial perturbation can develop into runaway instabilities in the form of avalanches or cascading processes. Considering a memoryless branching process as a mean-field approximation it is easy to reproduce a specific power-law distribution with exponent 3/2 [3]. We use more realistic and ad-hoc physical models of avalanche dynamics, still at a mathematical conceptual level, to assist us both in our understanding of the physical phenomena and in the fitting of empirical stochastic point-process models. The outcoming avalanche processes display fractal scale-free phenomena, finite size effects, regime transitions and inter-event correlations which are considered in the statistical and hazard models. We focus on some particular applications to natural and anthropogenic seismicity, material failure, structural and ferroic phase transitions.

References:

About noise, transients, and scaling laws close to saddle-node bifurcations

Authors:
• Josep Sardanyés (jsardanyes@crm.cat) Centre de Recerca Matemàtica

Abstract: In this talk we will present recent results about the role of demographic (intrinsic) noise in delayed transitions occurring at the vicinity of saddle-node bifurcations. Transient times typically become longer close to bifurcations in deterministic dynamical systems. They also obey to well-defined power laws which relate the duration of transients with how far the system is from the bifurcation. However, how intrinsic noise, relevant in small populations, shapes the length of these transients and their associated scaling laws is poorly understood. We have addressed this question by means of analytical work on simple stochastic models, extensive numerical simulations, and a Hamiltonian approach explaining the changes in the shape of the scaling functions for transient times.
In silico assessment of targeted combination therapies: Bivalent chromatin as a case study

Authors:

• Tomás Alarcón (talarcon@crm.cat) ICREA – Centre de Recerca Matemàtica

Abstract: Tumour cell heterogeneity is a major barrier to the efficient design of targeted anti-cancer therapies. A diverse distribution of phenotypically distinct tumour-cell subpopulations prior to drug treatment predisposes to non-uniform responses, leading to the elimination of sensitive cancer cells whilst leaving resistant subpopulations unharmed. Few strategies have been proposed for quantifying the variability associated with individual cancer-cell heterogeneity and minimizing its undesirable impact on clinical outcomes. Here, we report a mathematical model that allows the rational design of combinatorial therapies involving epigenetic drugs against chromatin modifiers. We have formulated a stochastic model of a bivalent transcription factor that allows us to characterise three different qualitative behaviours, namely: bistable, high- and low-gene expression. Comparison between analytical results and experimental data determined that the so-called bistable and high-gene expression behaviours can be identified with undifferentiated and differentiated cell types, respectively. Since undifferentiated cells with an aberrant self-renewing potential might exhibit a cancer/metastasis-initiating phenotype, we analysed the efficiency of combining epigenetic drugs against the background of heterogeneity within the bistable sub-ensemble. Whereas single-targeted approaches mostly failed to circumvent the therapeutic problems represented by tumour heterogeneity, combinatorial strategies fared much better. Our theoretical framework provides a coherent basis for the development of an in silico platform capable of identifying the epigenetic drug combinations best suited to therapeutically manage non-uniform responses of heterogeneous cancer cell populations.
MS16. PARTIAL DIFFERENTIAL EQUATIONS AND HOMOGENIZATION
Minimization and maximization of the first eigenvalue for a two-phase material

Authors:
• Juan Casado-Díaz (jcasadod@us.es Facultad de Matemáticas, C. Tarfía, Sevilla 41012, Spain) University of Sevilla

Abstract: We consider two diffusion materials (electric, thermic,...) represented by their diffusion constants in a bounded domain Ω ⊂ ℝ^N. We are interested in filling Ω with a mixture of these materials in order to maximize or minimize the first eigenvalue of the corresponding diffusion operator with homogeneous Dirichlet conditions. It is known that this type of problems has no solution in general and therefore that it is necessary to work with a relaxed formulation which can be obtained using the homogenization theory. We study the optimal conditions for this relaxed formulation, we get some equivalent formulations and we obtain some regularity results for the solution. As a consequence we show that if Ω has a connected boundary, then the unrelaxed formulation of the maximization of the first eigenvalue has a solution if and only if Ω is a ball. For the minimization the unrelaxed problem never has a solution.

References:
An asymptotic model by homogenization for elastic beams

Authors:
• Juan Casado-Díaz (jcasadod@us.es) Universidad de Sevilla
• Manuel Luna-Laynez (mllaynez@us.es) Universidad de Sevilla
• Antonio Pallares-Martín (ajpallares@us.es) Universidad de Sevilla

Abstract: We study the asymptotic behavior of the solution of the linear elasticity system in a thin beam of thickness $\varepsilon > 0$, when $\varepsilon$ tends to zero, imposing very weak assumptions on the elasticity tensors. We assume that the tensors vary with $\varepsilon$ arbitrarily (they have not to be periodic nor be defined from a fixed tensor by a change of variables), they are uniformly elliptic but non-uniformly bounded. We impose that the norm in $L^\infty$ of the elasticity tensors is an infinitesimal of $1/\varepsilon$ and the norm in $L^1$ is bounded. We do not ask them for any hypotheses of homogeneity or isotropy. These assumptions allow us to consider elastic beams made by mixtures, at a microscopic level, of several materials with very different characteristics (high-contrast homogenization problems). Passing to the limit by using homogenization theory, we obtain an asymptotic model with a system of linear equations in dimension one (ordinary differential equations). The solution of this general model provides an approximation of the behavior of the thin elastic beam which consists of the sum of a Bernoulli-Navier’s deformation plus a torsion term (an infinitesimal rotation around the axis of the beam). Although any equi-integrability property is assumed for the elastic tensors in the thin beams, the limit tensor in the asymptotic model is in $L^1$, i.e. it does not contain any measure supported on sets with null Lebesgue measure.

References:


Multiple positive solutions for an elliptic Kirchhoff equation

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• David Arcoya (darcoya@ual.es) Universidad de Granada
• José Carmona (jcarmona@ual.es) Universidad de Almería
• Pedro J. Martínez-Aparicio (pedroj.ma@ual.es) Universidad de Almería

Abstract: We are concerned with the Kirchhoff elliptic problem

\[
\begin{aligned}
- \left( 1 + \gamma G' \left( \|\nabla u\|_{L^2(\Omega)}^2 \right) \right) \Delta u &= \lambda f(u) & \text{in } \Omega, \\
\quad u &= 0 & \text{on } \partial \Omega,
\end{aligned}
\]

where \( \Omega \) is an open, bounded subset of \( \mathbb{R}^N \) (\( N \geq 3 \)), \( f \) is a locally Lipschitz continuous real function, \( f(0) \geq 0 \), \( G' \in C(\mathbb{R}^+) \) and \( G' \geq 0 \). For large \( \lambda \) we prove multiplicity of positive solutions with \( L^\infty(\Omega) \) norm between two consecutive zeroes of \( f \).

References:

On optimal potential problems

Authors:
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• Faustino Maestre (fmaestre@us.es) Universidad de Sevilla

Abstract: In this work we present an existence result for a variational optimal control problem governed by an elliptic partial differential equation of a Schrödinger type where the control is the coefficient of zero order given by a potential $V$. We will study the case where the problem is stated in a bounded domain $\Omega$ and the right-hand side of the state equation has not fixed sign and the extension to unbounded domains. Some necessary conditions for optimality are showed and several numerical experiments.

References:


Un sistema elíptico no lineal en espacios de Sobolev-Orlicz anisó tropos: análisis y simulación numérica

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Abstract: Se establece un resultado de existencia de solución de capacidad de un sistema elíptico, no lineal y acoplado formulado en espacios de Orlicz-Sobolev anisó tropos. Las incógnitas de este sistema son la temperatura en un material semiconductor y el potencial eléctrico. Se trata de una generalización del denominado problema del termistor.

La aproximación numérica también es analizada mediante un algoritmo de mínimos cuadrados combinado con la versión de Polack-Ribiére del método del gradiente conjugado. Finalmente, se presentan algunos resultados de distintas simulaciones numéricas.

References:


MS17. Mathematics in Industry and Organizations
Quality classification of recycled plastics

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Abstract: The classification of the quality of a product is a key issue for both manufacturers and consumers. In particular, being able to quantify whether a certain material, such as plastic, is of better or worse quality, is a problem that until now has been solved in terms of experience and from a subjective perspective by means of personnel expert in the classification procedure. The present work describes various algorithms that have been used within a project with the company CADEL DEINKING, S. L. in order to obtain an automatic procedure leading to the quantification (even approximate) of the quality of the final product (plastic) obtained by the company.
Data fitting in a portable post-harvest system. Removal of attached ink in plastic.

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Abstract: In this talk we present two collaborations with the industry. The first one consists on a portable post-harvest system that required a robust curve fitting algorithm to be implemented in a very modest hardware. The system is developed by the company CITROSOL, S.L. and is able to provide an in-situ measurement of the concentration of certain chemicals with results similar to the official reference laboratories. This immediately provides the producer with valuable information on compliance with legal requirements on residue levels for the distribution of fruit and vegetable products.

The second one aims to develop a digital twin for a cleansing system that removes attached ink from plastic prior to recycling. The company CADEL DEINKING, S. L. has developed a unique process that allows the ink printed on plastics to be removed before obtaining the recycled material. This gives the product a high added value, since the printing ink damages the recycled product both visually and in mechanical properties. The result of the process is an ink-free plastic with a quality very similar to virgin plastic. Our collaboration consists on the development of a digital twin of the system.
Machine Learning techniques for marketing using social login data.

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Abstract: Social login facilitates the registration of users on websites and apps using information coming from their social network accounts. In return, user data is gathered by the hosting company. This information has a myriad of potential marketing applications, such as targeting customers that are likely to purchase a certain product or identifying users with similar interests. In this talk, we present some Machine Learning techniques applied to a real scenario that leverage social login data to produce better marketing campaigns and improve the target customers’ selection. Problems treated include lookalike modeling, personality traits prediction, and communities detection. The focus will be on the mathematical background of these approaches and their benefits for the marketing industry.
Mesoscopic modelling and simulation of espresso extraction

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Abstract: Espresso extraction is a very complex process involving flows through a porous medium undergoing geometric changes, solubilization of many hydrophilic substances, emulsification of insoluble coffee oils, suspension of solid coffee cell-wall fragments (‘fines’), and CO2 degassing and supersaturation. In this research we investigate the effects of changing geometry on the extraction. We devise mesoscopic models in the framework of smoothed particle hydrodynamics (SPH) to model fine migration, particle erosion, and particle swelling. It is found that fine migration leads to a transient permeability for the porous matrix, and results in an increase of compound content in the cup[1]. Particle erosion induced by the mechanical force of flow leads to channelisation phenomenon and, if the erosion is pressure-dependent, a heterogeneity in the filtration direction will also be induced. As a consequence, the flow is also heterogeneous and the extraction is unbalanced in the matrix[2]. Particle swelling affects both the intra-granular and inter-granular transport of the soluble compounds. A very small degree of swelling will significantly enhance the compound content in the cup, altering the taste and aroma of the espresso[3].

In addition to the SPH models, we also devise continuum models to investigate the erosion and swelling process in a porous medium. Results similar to those from SPH are obtained using these models. Finally, the possibility to target real systems using X-ray microCT data of coffee capsules will also be discussed.

References:


Optimal management of a routing protocol in a call center for staff dimensioning

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Abstract:
Queuing theory is a powerful tool of operations research theory for optimizing the staff dimensioning in a call center. When the service is composed of multiple queues associated to different categories of incidences, then, routing strategies with multiskill agents and linear programming can be combined in order to get a fluent and effective management of the service and to reach adequate standards of quality. An essential condition for the mathematical model to be efficiently applied is to provide a good data sets, especially the incoming and outcoming rates. Some algorithms for the database manipulation has been developed to guarantee the quality of the calculus of the parameters used for running simulations that reflect real-world situations. The research project has been carried out together with a distribution company and the model developed here has been integrated in its internal call center.

References:
Numerical simulations for tsunami preparedness risk assessment in Andalusian coast. Indra-Edanya group collaboration.

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Abstract:
The use of high-resolution topobathymetric data for tsunami simulation can provide relevant information on the extent of the inundation caused by it. In addition, the consideration of complementary data such as the type of buildings, the location of essential infrastructures, the type of roads, airports, hospitals, etc. allow to identify the most vulnerable areas, estimate the expected damage to these infrastructures, limiting the affected and evacuation area, defining evacuation routes, adapting emergency plans and evaluating the associated vulnerability in the different areas.

With this objective, the Spanish General Directorate of Civil Protection and Emergencies (CENEM) triggered the Copernicus Emergency Management Service (CEMS) Risk and Recovery Mapping to develop tsunami risk analysis in the urban environments along the Andalusian Coast at very high resolution.

In this talk, we will present the results of the INDRA-UMA collaboration in these Copernicus Activations, emphasizing the use of the Tsunami-HySEA numerical model, developed by the Edanya group, from the University of Málaga, which is specifically designed for simulations of earthquake-generated tsunamis on the real earth.

This is a very important milestone that has been achieved, since it is the largest extension covered at high resolution and evaluated for tsunami risk ever done before.

References:
MS18. Geometric flows and PDEs in Geometry
Generalized Ricci Flow

Authors:
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Abstract:
The generalized Ricci flow equation is a geometric evolution equation which has recently emerged from investigations into mathematical physics, Hitchin’s generalized geometry program, and complex geometry. The generalized Ricci flow can regarded as a tool for constructing canonical metrics in generalized geometry and complex non-Kähler geometry, and extends the fundamental Hamilton/Perelman theory of Ricci flow. In this talk I will give an introduction to this topic, with a special emphasis on examples and geometric aspects of the theory. Based on joint work with Jeffrey Streets [1].

References:
SKT structures and the pluriclosed flow on solvmanifolds.

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Abstract:
The pluriclosed flow is a geometric flow that evolves strong Kähler with torsion Hermitian structures —SKT for short— on a given complex manifold. The aim of this talk is to discuss the asymptotic behaviour of the pluriclosed flow in the case of invariant structures on nilpotent and almost abelian Lie groups.

This talk is based on joint works with Ramiro Lafuente (The University of Queensland) and Marina Nicolini (Universidad Nacional de Córdoba).
Supersymmetric spinorial flows on three-dimensional Cauchy hypersurfaces

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Abstract:
In this talk I will introduce the audience to the Cauchy problem for globally hyperbolic supersymmetric configurations. I will first introduce the notion of globally supersymmetric configuration as a solution to the supergravity spinorial equations. Then, I will briefly recall the theory of spinorial polyforms associated to bundles of irreducible real Clifford modules, which provides a convenient geometric framework to study first-order differential spinorial equations and their moduli. Using this framework, I will provide a novel geometric description of the initial value problem and evolution flow of generic globally hyperbolic supersymmetric configurations and I will apply it to the simplest class of supersymmetric configurations, namely globally hyperbolic Lorentzian four-manifolds equipped with a parallel spinor, also known as pp-waves, whose evolution flow defines the parallel spinor flow. I will classify all compact three-manifolds admitting admissible initial data to the parallel spinor flow, proving that they consist of torus bundles over $S^1$ canonically equipped with a possibly non-proper locally-free action of $\mathbb{R}^2$. In addition, I will prove that despite the fact that Lorentzian metrics admitting parallel spinors are not necessarily Ricci flat, the parallel spinor flow preserves the vacuum momentum and Hamiltonian constraints and therefore the Einstein and parallel spinor flows coincide on common initial data. Using this result, I will provide an initial data characterization of parallel spinors on Ricci flat Lorentzian four-manifolds. Finally, I will explicitly solve the left-invariant parallel spinor flow on simply connected Lie groups. Interestingly enough, some of these flows define families of $\eta$-Einstein cosymplectic structures and solutions to the left-invariant Ricci flow in three dimensions.

References:
The Chern-Ricci flow on Inoue surfaces

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Abstract:
We are interested in some analytic problems concerning the geometry of the Chern connection of Hermitian manifolds, e.g.: symmetries of the curvature tensor [3]; the existence of metrics with constant Chern-scalar curvature [1]; the generalizations of the Kähler-Einstein condition to the non-Kähler setting [2, 4]; the convergence of the normalized Chern-Ricci flow on compact complex surfaces [5]. Concerning the latter problem, in particular, we prove that every Gauduchon metric on an Inoue-Bombieri surface admits a strongly leafwise flat form in its $\partial\bar{\partial}$-class. Using this result, we deduce uniform convergence of the normalized Chern-Ricci flow starting at any Gauduchon metric on all Inoue-Bombieri surfaces.

References:


Linearity of homogeneous solutions to degenerate elliptic equations in dimension three

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Abstract:
Given a linear elliptic equation \( \sum a_{ij} u_{ij} = 0 \) in \( \mathbb{R}^3 \), it is a classical problem to determine if its degree-one homogeneous solutions \( u \) are linear. This is a question motivated by global surface theory. The answer to this problem is negative in general, by a construction of Martinez-Maure. In contrast, the answer is affirmative in the uniformly elliptic case, by a theorem of Han, Nadirashvili and Yuan, and it is a known open problem to determine the degenerate ellipticity condition on \( (a_{ij}) \) under which this theorem still holds. In this paper we solve this problem. We prove the linearity of \( u \) under the following degenerate ellipticity condition for \( (a_{ij}) \), which is sharp by Martinez-Maure example: if \( K \) denotes the ratio between the largest and smallest eigenvalues of \( (a_{ij}) \), we assume \( K|_O \) lies in \( L^1_{\text{loc}} \) for some connected open set \( O \subset S^2 \) that intersects any configuration of four disjoint closed geodesic arcs of length \( \pi \) in \( S^2 \). Our results also give the sharpest possible version under which an old conjecture by Alexandrov, Koutroufiotis and Nirenberg (disproved by Martinez-Maure’s example) holds. This is a joint work with P. Mira.

References: